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ON THE DENSITY AND SURFACE BRIGHTNESS PROFILES IN GLOBULAR STAR CLUSTERS

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SUMMARY: A model of mass distribution, applicable to globular star clusters and proposed earlier, is reconsidered. It is shown that it can be related to the well-known Plummer-Schuster formula, and the equations yielding its dimensionless parameters are given. The corresponding surface density is calculated numerically. It is indicated that in a general case the surface density should not be proportional to the surface brightness and a more adequate formula relating these two quantities is proposed.

Key words. globular clusters: general

1. INTRODUCTION

One of the most important questions in studying the globular star clusters certainly concerns the mass distribution within them. In this context, usual approximations are that a typical globular cluster (GC) is in a steady state and spherically symmetric. Then the most frequently used description of the mass distribution, is the one following from King's (1962) formula. However, there are two drawbacks of this formula. The first is that it does not yield any analytical expression for the cumulative mass, i.e. the potential. The second is that the mass segregation is not taken into account, more precisely, it is removed by means of an unrealistic assumption that all the stars belonging to a GC have the same mass. There have been attempts to eliminate the latter drawback by relatively complicated procedures based on stellar dynamics (e.g. Da Costa and Freeman 1976, Davoust 1977). The usual approach has been to consider the mass distribution of stars as discrete, involving various classes of star masses. On the contrary, the standpoint of the present authors as based on an earlier paper (Ninković 1996) is that the more realistic assumption of a continuous mass distribution of stars also offers possibilities in solving of this problem.

The question concerning the formula for the potential is also considered here, and a particular formula, proposed earlier (Ninković 2003), is reexamined in the light of its application to GCs. We give in the present paper a general procedure how the observational data (profile of surface brightness) can be corrected in order to find the mass distribution within a GC where the phenomenon of mass segregation is taken into account. We also examine the application of the alternative formula yielding mass distribution.

Table 1. Values of dimensionless parameters of eq. (1)

λ	∞	300	200	150	100	80	60	40	20
α	0	0.0025	0.008	0.009	0.013	0.019	0.025	0.038	0.075
κ	1	0.99998	0.99997	0.99994	0.99991	0.9998	0.9997	0.9993	0.997
λ	10	8	5	3	2.7	1.5	0.85	0	
α	0.15	0.185	0.29	0.45	0.5	0.7	0.9	1	
κ	0.9899	0.985	0.96	0.90	0.88	0.73	0.47	0	

2. MASS DISTRIBUTION WITHIN A GLOBULAR CLUSTER

In the paper by one of the present authors mentioned above (Ninković 2003), a particular mass distribution, applicable to GCs as well, was studied. However, additional remarks are necessary and they will be given in the present paper.

As a first step, the formula for potential will be rewritten and rediscussed.

$$\Pi = \frac{\kappa G \mathcal{M}}{(r^2 + r_c^2)^{1/2} - \alpha r_c} \ . \qquad (1)$$

Here G is the universal gravitational constant, $\mathcal{M}$ is the total mass of the stellar system (GC), r_c is its scale length (core radius), κ and α are two dimensionless, non-negative, parameters and r is the variable (distance to the centre). The interval covered by the dimensionless parameter α is $\alpha \in [0, 1)$. In the limiting case $\alpha = 0$, this mass distribution becomes that of the Schuster, or Plummer type and then the density vanishes in the infinity. This aspect was not discussed in the earlier paper (Ninković 2003). For this reason, in the present paper it will be emphasized that such a result corroborates the mass distribution with the corresponding potential expressed by Eq. (1). Indeed, if the limiting radius becomes infinite, then the factors contributing to the limitation of the size of a GC (tidal action, etc.) are weak and the cluster may appear as infinite. In our opinion this is more realistic than the situation with the models proposed by King (1962) of the one often referred to as the modified Hubble-Reynolds model (e.g. Ninković 1998) because they yield an infinite total mass when the limiting radius tends to infinity. However, if $\alpha > 0$, the limiting radius is finite. Also the behaviour of the other dimensionless parameter, κ, follows that of α. If $\alpha = 0$, then κ is equal to 1 as should be the case for the Plummer-Schuster distribution. When $\alpha > 0$, then κ becomes smaller than 1. We give here two equations which show how both α and κ can be calculated for a given ratio of the limiting radius r_l to the scale length r_c. One of them was presented in the earlier paper (Ninković 2003), but it will be given here in an improved form in which the relation between r_l/r_c and α is more clearly seen; for practical reasons the ratio of the two radii will be denoted as λ. The equation yielding κ is obtained from the condition for the potential: at the limiting radius, the potential (1) equals that of a point mass at the same distance. The corresponding formulae are

$$\alpha = \sqrt{\lambda^2 + 1} - \frac{2}{\frac{3\sqrt{\lambda^2+1}}{\lambda^2} - \frac{1}{\sqrt{\lambda^2+1}}} \ ; \qquad (2)$$

$$\kappa = \frac{\sqrt{\lambda^2 + 1} - \alpha}{\lambda} \ . \qquad (3)$$

In order to see the meaning of (2) and (3) clearly enough, the dependences of α and κ on λ are presented in Table 1.

With regard to the values of the ratio r_l/r_c for the GCs of our Galaxy, the small values of α (less than 0.1) are more interesting. Thus the role of the dimensionless parameter α can be better seen: it appears as a correction to the Plummer (or Schuster) formula for the potential assuming that the density vanishes at a finite distance from the centre. In this way, two in principle different kinds of density distribution are unified, and it is thus easier to understand why in the history of studying GCs (mostly those belonging to our Galaxy) both types of density distribution have been proposed as correct descriptions of the real state (see Ninković 1998 and the references therein, also Jefferys 1976). Later on, the point of view that GCs should be modelled as systems with finite radius became generally accepted because the observations have shown that the values of the limiting radii can be indicated sufficiently reliably.

The values of the limiting radii for GCs belonging to our Galaxy, found observationally, are viewed usually as consequences of the tidal forces. However, the relation connecting the limiting radius of a GC and the tidal limits is not a simple one, among other reasons, due to the rather complicated galactocentric motion of GCs. The observed radii for many GCs in the Galaxy seem to be smaller than the corresponding dynamical radii attributed to the tidal forces (e.g. Brosche et al. 1999). In addition, a correlation between the radius of a GC and its galactocentric distance has been established (e.g. Zakhozhaj 2005). Due to these factors a GC can be understood as an aggregate of stars with a finite mass where the radius surrounding the bulk of its stars is limited rather strongly also by the forces of the galactic tidal field. As a consequence, a gradual and regular density decrease, typical of the Plummer-Schuster case, gives way to an abrupt one, and the mass distribution is to be described by means of a model with a well defined outer radius. Thus, it becomes possible to explain why one introduces and prefers the model based on the gravitational potential given by Eq. (1).

3. SURFACE-DENSITY AND SURFACE-BRIGHTNESS PROFILES WITHIN A GLOBULAR CLUSTER

Unfortunately, the density, corresponding to the potential given by (1), yields no analytical expression for the surface density. This is, obviously, a drawback in comparison with King's model where one obtains analytical expressions for both volume and surface densities.

The fact that the mass distribution (1) is close to the Plummer-Schuster one, allows to obtain a numerical solution for the surface density relatively easily. In principle, one uses the solution analogous to the one for the Schuster case (e.g. Ninković 1998) to introduce afterwards a correction yielding the vanishing of the surface density at a finite distance to the cluster centre in projection. The correction is not large if α is sufficiently small, and this is just the case of special interest here (see Table 1).

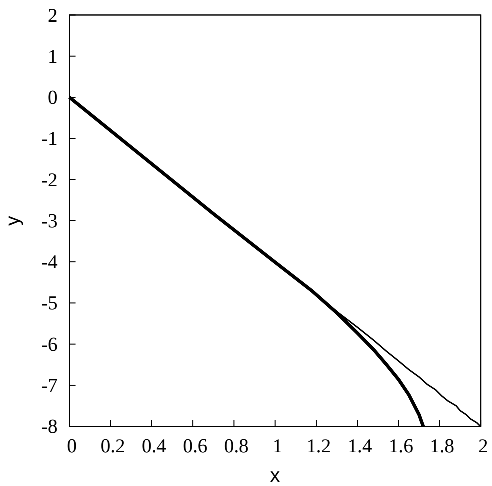

Fig. 1. *Logarithmic dependence of the dimensionless surface density y, $y = log\frac{\sigma}{\sigma(0)}$, on the dimensionless radius in projection x, $x = log\frac{\tilde{r}}{r_c}$; the thick line corresponds to the Eq. (1), with* $\alpha = 0.025$, *whereas the thin one pertains to the classical Plummer-Schuster case with the same radius r_c.*

Fig. 1 gives the logarithmic plot for the dependence of the surface density on the radius in projection. This case corresponds to a small α (α=0.025, since the other two dimensionless model parameters depend on it, their values can be found in Table 1) and it shows how the slope of the classical curve corresponding to the Schuster law can be corrected to obtain a curve in which the surface density vanishes at a finite distance.

However, if we want to test the surface-density profile presented in Fig. 1 observationally, we must take into account that the observable, in fact, is not the surface density, but the surface brightness. The most simple solution is to assume that these two quantities are proportional to each other. In our opinion this assumption is not correct. Therefore, we shall give here a more adequate relation.

With regard to the assumption concerning the steady state and spherical symmetry mentioned above, the integrated apparent magnitude of a GC will be given as

$$m_{\rm int} = 2\pi \int I(\tilde{r})\tilde{r}d\tilde{r}$$

where $\tilde{r}$ is, as previously, the distance to the cluster centre in projection (or distance to the line of sight) and $I(\tilde{r})$ is the surface brightness. The integral is, clearly, taken over the entire solid angle occupied by a cluster.

In the present paper it is also assumed that the luminosity of a GC is due to its stars only. Thus, the surface brightness can be represented as a product - $I = \tilde{n}\bar{m}_{\rm nor}$ - where $\tilde{n}$ is the surface number density of stars and $\bar{m}_{\rm nor}$ is the normalized mean apparent magnitude of a star. This mean value must be normalized because apparent magnitude is not an additive quantity. The normalization means dividing the average apparent magnitude by the total number of cluster stars. Both the surface number density and the normalized mean apparent magnitude are functions of $\tilde{r}$, just as I. On the other hand, due to the lack of interstellar matter assumed above, an analogous relation is valid for the surface (mass) density - σ, $\sigma = \tilde{n}\bar{\mathfrak{m}}$, where $\bar{\mathfrak{m}}$ is the mean mass of a star, also a function of $\tilde{r}$. From these two relations it follows that

$$\sigma = \frac{\bar{\mathfrak{m}}}{\bar{m}_{\rm nor}} I \; .$$

From this formula it is clearly seen that the surface density will be proportional to the surface brightness, only if the ratio of the mean mass to the normalized mean apparent magnitude is constant, which, under the physical conditions, means that both are constant. However, within GCs the phenomenon known as mass segregation can take place and due to it the mean mass and the normalized mean apparent magnitude are expected to become variable. Therefore the present authors insist on this relation as more correct than a simple assumption of proportionality between the two quantities.

Thus King's formula could be accepted as complementary to the model discussed in the previous section. Namely, there is a possibility that it offers a correct description of the surface brightness, but in view of the comment concerning the relation between the surface brightness and surface density, an a posteriori correction for the purpose of conversion of the profile of the surface brightness into that of surface density becomes necessary. Therefore, any alternative formula describing the mass distribution within a GC is welcome.

4. CONCLUSION

In the present paper, a model of mass distribution proposed in Ninković (2003) is reexamined. Its connection to the Plummer-Schuster model is shown, and also its applicability to GCs is considered. Since the surface density, in fact, is not an observable, but this is the surface brightness, a formula relating the two quantities is derived. This formula, according to the present authors, is more correct than a simple assumption of a mere proportionality between the surface density and the surface brightness.

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О ЗАВИСНОСТИ ГУСТИНЕ И ПОВРШИНСКОГ СЈАЈА У ЗБИЈЕНИМ ЗВЕЗДАНИМ ЈАТИМА

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Поново се разматра један модел расподеле масе, предложен раније, који се може применити на збијена звездана јата. Показана је његова веза са добро познатом Пламер-Шустеровом формулом и дају се једначине за добијање његових бездимензионих параметара. Одговарајућа површинска густина је израчуната нумерички. Указује се да у општем случају површинска густина не треба да буде пропорционална површинском сјају и изведена је једна адекватнија формула која повезује ове две величине.