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ON THE DENSITY WITHIN THE DARK-MATTER CORE IN OUR GALAXY

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SUMMARY: Assuming that the disc of our Galaxy, the Milky Way, obeys the classical exponential law, that it is maximal and the Sun is rather far from the maximum of its circular velocity, one finds that, most likely, the galactic corona (subsystem containing the dark matter) has a nearly constant density within its core which contains the position of the Sun. The approach applied in the present paper is local, i.e. quantities characterizing the solar neighbourhood are treated. The assumptions and the result could explain why the ratio of the moduli of the Oort constants is expected to exceed the value of 1.0 which corresponds to the locally flat rotation curve of the Milky Way.

Key words. solar neighbourhood – dark matter

1. INTRODUCTION

Dark matter (DM) is generally known today as existing, if not in all, then in many galaxies. Our own Galaxy, the Milky Way (MW), is also known to be dominated, as for its total mass, by DM. The subsystem populated by DM is usually referred to as dark halo or corona. There is an important question concerning the coronae of galaxies: is their density nearly constant within the core or it has a cusp? The cosmological arguments based on simulations are in favour of a cusp (e.g. Navarro, Frenk and White 1996, 1997), but the evidence coming from observational studies of individual galaxies indicates that their coronae should have cores of nearly constant density (e.g. Salucci 2001). Such a problem is expected, clearly, to affect MW, too. For the purpose of answering this question most frequently a global model of a galaxy based on its rotation curve is used. In the case of MW one can also use the data concerning the solar neighbourhood (local data, as they are often called). The present paper is namely characterized by an approach of this type.

The Oort constants, A and B, are defined as

$$A = -\frac{1}{2} R_{\odot} \frac{d\omega_c}{dR}(R_{\odot}) \tag{1}$$

$$B = A - \omega_c(R_{\odot}) \ , \tag{2}$$

where ω_c is the angular circular velocity, more precisely the ratio between the circular velocity and the corresponding distance, whereas R is the distance to the axis of MW rotation. The ratio of their moduli seems to be rather well known; the corresponding formula will be given in the next Section.

In addition, the evidence that the disc of MW is maximal (dominant in contribution to square of local circular velocity) has grown significantly in recent times. This circumstance appears as an essential constraint to the corona contribution to the local square of circular velocity; also if it is taken into account that the scale length of the disc is rather well known, then, as will be shown below, it is possible to form a system of equations yielding the density slope for the corona within its core.

2. FORMALISM

To begin with, one treats the circular velocity. Clearly, the corresponding angular velocity is obtained as the quotient of the circular velocity and the distance at which it is taken. The square of this angular velocity has the same dimension as the second radial derivative of the gravitational potential Π. Following Ninković (1987) the ratio of the latter one to the former one is introduced here, where the two quantities are taken at the same position; in the present paper this is the position of the Sun.

In the case of MW there are several contributors to the galactic potential. Therefore, for its second derivative the following will be valid

$$\frac{\partial^2 \Pi}{\partial R^2} = \sum_i \frac{\partial^2 \Pi_i}{\partial R^2} \quad . \tag{3}$$

With regard to the definition given above, the second potential derivative for each contributor can be rewritten as $\gamma_i \omega_{ci}^2$. If the weights of contributing subsystems are introduced, namely $w_i = \omega_{ci}^2/\omega_c^2$, the following expression holds

$$\gamma = \sum_i \gamma_i w_i \quad . \tag{4}$$

It should be noted again that the dimensionless quantity γ depends on the position (more simply, on the distance to the galactic rotation axis since the present study is limited to the galactic plane) at which it is taken, however, for convenience, $(R_\odot)$ will be omitted since here in most cases one deals with the local values of γ, i.e. of a γ_i. In addition, γ, as a dimensionless quantity, can be easily related to the ratio of the Oort-constant moduli by means of the following formula (Ninković 1992)

$$\gamma = \frac{3A + B}{A - B} \ , \tag{5}$$

i.e.

$$\gamma = \frac{3\alpha - 1}{\alpha + 1} \ , \tag{6}$$

where

$$\alpha \equiv \frac{A}{|B|} \ . \tag{7}$$

Eq. (4) is the main equation in the present paper, but there is an additional constraint. Namely, the normalized sum of the weights, clearly, must be unity.

It is almost generally accepted that there are three essential contributors to the potential of MW: the (thin) disc, the bulge and the corona; the mass of the thick disc is too small ($\sim 10^9\ \mathcal{M}_\odot$) to be taken into account. This assumption will be followed in the rest of the present paper, therefore also in solving Eq. (4).

With regard to Introduction Eq. (1) will be solved in the two parameters concerning the corona - its weight and the coefficient γ_c.

As it is well known, the International Astronomical Union (IAU) recommends values of 8.5 kpc and 220 km s^{-1} for the distance of the Sun to the axis of MW rotation and the corresponding circular velocity, respectively. Though these both figures have been subject to criticism, they are today generally accepted as sufficiently close to reality. There is also a paper by Feast and Whitelock (1997), cited very often as the source for the values of the two important quantities mentioned above, where the same amounts as those recommended by IAU are given. Therefore, they are also taken in the present paper. In this way the angular velocity becomes known.

The bulge is a subsystem of MW concentrated rather closely to the centre of this galaxy and not very flattened. In view of this its effective boundary may be represented as a sphere with radius significantly smaller than $R_\odot$. In other words at the position of the Sun the potential of the bulge would be that of point mass according to Newton's second theorem (e.g. Binney and Tremaine 1987, p. 34). Then one will have $\gamma_b = 2$. The total mass of the bulge is thought to be about $10^{10}\ \mathcal{M}_\odot$, which is also assumed here. In this way the weight of the bulge in Eq. (4) is easily calculated to be 10%.

The existing evidence concerning the galactic disc (e.g. Kuijken and Gilmore 1991, Flynn and Fuchs 1994, Holmberg and Flynn 2000, Sackett 1997) yields a total mass of about $6\times10^{10}\ \mathcal{M}_\odot$ and a most likely scale length of 2.5 kpc. In the present paper, the disc is assumed to be exponential characterized by these two values. The dependence of the circular velocity on the distance to the centre for an exponential disc (Freeman 1970) makes it possible to calculate the corresponding weight provided that the ratio of $R_\odot$ to the disc scale length is known. In this way, one finds the weight of the disc in (4) to be 75%. Clearly, the weight of the corona is known now to be 15%.

The value for γ_d is found by applying the dependence mentioned above (Freeman 1970). It can be determined from the slope of the curve describing the circular velocity of the disc as function of the distance at the given position, more precisely ratio of $R_\odot$ to the disc scale length. With the values assumed above, this ratio is equal to 3.4, a value for which the dependence (Freeman 1970) yields γ_d to be about 1.5, or in other words the product concerning the disc in (4) will be equal to 1.1. The quantity γ from the left-hand side of (4) can be obtained from the ratio of the Oort-constants moduli. It is noticeable that more recent papers (e.g. Feast and Whitelock 1997, Dehnen and Binney 1998) suggest a value significantly smaller than 1.5 which was once the official value recommended by IAU. If the circular-velocity curve were flat at the Sun, the ratio of the Oort-constants moduli would be equal to 1.0. Feast and Whitelock (1997) suggest a somewhat higher value, of about 1.17, yielding $\gamma = 1.15$ according to the relation connecting these two dimensionless quantities (Ninković 1992). If this value is substituted in (4), then for the corona one will have γ_c about -1. If borne in mind that for a homogeneous sphere it is exactly $\gamma = -1$, such a result means that within the core of the corona of MW the density is nearly constant. A model describing the mass distribution within the corona having γ_c exactly equal to -1 at the centre would be acceptable. Besides, the calculations of the dependence of the circular velocity on distance (rotation curve) for MW where the corona core is assumed to have exactly constant density have already been carried out (e.g. Nikiforov et al. 1999). The parameters of the subsystems used in Eq. (4) are given in Table 1.

Table 1. Parameter values applied in Eq. (4)

	bulge	disc	corona
w_i	0.1	0.75	0.15
γ_i	2.0	1.47	-1.0

3. DISCUSSION

The basic assumptions used in the calculations (Eq. (4)) are that the disc is maximal and that its scale length is significantly smaller than the galactocentric distance of the Sun (the corresponding ratio about 3.5). These two assumptions determine the two quantities - w_d and γ_d - appearing on the right-hand side of (4); more precisely, the former one determines the value of w_d and the latter one the value of γ_d, also, if the disc is maximal, then the weights of other subsystems cannot be significant. For instance, if the disc is maximal, then the total mass of the bulge cannot be large because a sufficiently large mass of the bulge would increase its weight significantly. In addition, with regard to our understanding of the bulge as a subsystem situated in the vicinity of the galactic centre its mass concentrated within the Sun must be total. In this way the value for γ_b is fixed.

As for the disc itself, in the present paper the classical exponential model is assumed for its mass distribution. This model has two parameters: the central surface density and the scale length. The former parameter is obtained from the local surface density of the disc. Therefore, the following comment holds. If the local values, in particular the local surface density of the disc, the galactocentric distance of the Sun and local circular velocity, are fixed as more reliable, then the resulting total mass of the disc will increase with the disc scale length decreasing. Since the disc contribution to the square of the local circular velocity is directly proportional to its total mass and inversely proportional to the scale length, it is clear that this contribution will increase in such conditions. True, there is a numerical factor (the solution of the corresponding Poisson equation - Freeman 1970), but its influence is weaker and the general conclusion that the weight of the disc (w_d in Eq. (4)) increases, when the total mass increases and the scale length decreases, is still valid. Thus the weight of the disc is affected by both local surface density and scale length, unlike γ_d which is affected by the scale length only.

The calculations based on Eq. (4) under the assumptions stated above yield a value close to -1 for the dimensionless quantity γ_c at the galactocentric position of the Sun. Such a value is typical for central parts of stellar systems resembling a homogeneous sphere. In the case of an alternative, say cuspy, model the central value would have a lower limit exceeding -1. As well-known examples of cuspy models, the ones proposed by Hernquist (1990) and Jaffe (1983) may be mentioned. Their lower limits for γ_c are 0 and 1, respectively. In general the cuspy models with a power law desribing the density behaviour within the cores, like those proposed by Dehnen (1993) and Tremaine et al. (1994), yield for γ near the centre values in accordance with the degree of that power law; if the cumulative mass, i.e. density, is described by means of a power law, then γ is constant. For this reason one has a lower limit of γ exceeding -1.

On the other hand, if the corona is spherical, as usually assumed - which does not mean that it is a perfect sphere (e.g. Samurović, Ćirković and Milošević-Zdjelar 1999) - its contribution to the circular velocity clearly determines the corona mass within the Sun. The value of its total mass can follow from a number of indirect pieces of evidence (e.g. Wilkinson and Evans 1999). In this way one can determine the fraction of the corona total mass within the Sun. This quantity can serve as a nice test for each corona model under examination because, as easily understood, knowing it one finds the other characteristic distances, for instance half-mass radius and so on. For instance, according to the two models with a cuspy mass distribution mentioned above - Hernquist (1990) and Jaffe (1983) - a significant fraction of the corona total mass should be contained within extremely large galactocentric distances, almost reaching that of the Andromeda Nebula. Owing to this, they are easily ruled out. This is an additional argument in favour of a mass model for the corona which yields nearly constant density within its core.

The maximality of the galactic disc has another consequence very easily seen from Eq. (4). Because of it the term on the right-hand side describing the disc becomes dominant and the value for γ is strongly affected by that for γ_d. However, the latter one, for the case of the exponential model, depends only on the ratio of $R_\odot$ to the scale length of the disc. Higher values for this ratio like, say 3.0-3.5, as assumed in the present paper, yield sufficiently large γ_d after which it becomes almost impossible to obtain equal moduli of the Oort constants (flat circular-velocity curve). In this way we can understand why the ratio of the Oort-constants moduli is, nevertheless, expected to be slightly greater than 1.0 as, for instance, found by Feast and Whitelock (1997). In addition, since γ_b and γ_c most likely have opposite signs and the corresponding weights are not much different, the sum of the first and last term on the right-hand side in (4) is approximately zero, so that we obtain a very simple approximative form for this equation

$$\gamma \approx \gamma_d w_d \ . \tag{8}$$

From this relation it is easily seen that for a maximal disc a γ_d value of about 1.5 implies approximately $\gamma = 1.15$. According to the formula (Ninković 1992) mentioned above, the corresponding ratio of the moduli of the Oort constants would be about 1.17. It is curious to note that the value for γ_d obtained here, based on that for the disc scale length assumed above, is close to a value corresponding to a ratio of the moduli of the Oort constants of 1.5 which was once that recommended by IAU (Hamburg GA in 1964). In addition, at that time it was thought that the gravitation field of the disc determined that of MW as a whole.

Among possible influences on the present results one can also mention the presence of an additional subsystem in Eq. (4). In particular, the thick disc and galactic (classical) halo are borne in mind. The former one has been already commented above. The latter one has not usually been taken into account in the calculations of the galactic potential due to its very small total mass (e.g. Carney et al. 1990), as well as to the fact that the halo is a rather loose subsystem. These properties of the halo appear as strong constraints to its weight; most likely it does not exceed 5%. In addition, all what is known about the spatial distribution within the halo indicates that γ_h should be closer to the corresponding value for the disc than to that for the bulge, thus $\gamma_d \approx 1.5$. If substituted in (4), this makes the correspoding product to remain within the error limits and, practically, nothing is changed in the present conclusions.

4. CONCLUSIONS

If the galactic disc obeys the exponential density law, is maximal and its scale length is sufficiently small (say 3-3.5 times smaller than the galactocentric distance of the Sun), then the density of the dark corona should be almost constant in the inner parts of MW which include also the position of the Sun. This has a consequence that the local slope of the circular-velocity curve for the disc much determines the ratio of the moduli of the Oort constants for which in such a situation a value of 1.0 or less has a low probability.

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REFERENCES

Binney, J. and Tremaine, S.: 1987, Galactic Dynamics, Princeton University Press, Princeton, New Jersey.
Carney, B.W., Latham, D.W. and Laird, J.B.: 1990, *Astron. J.*, **99**, 572.
Dehnen, W.: 1993, *Mon. Not. R. Astron. Soc.*, **265**, 250.
Dehnen, W. and Binney, J.J.: 1998, *Mon. Not. R. Astron. Soc.*, **294**, 429.
Feast, M. and Whitelock, P.: 1997, *Mon. Not. R. Astron. Soc.*, **291**, 683.
Flynn, C. and Fuchs, B.: 1994, *Mon. Not. R. Astron. Soc.*, **270**, 471.
Freeman, K.C.: 1970, *Astrophys. J.*, **160**, 811.
Hernquist, L.: 1990, *Astrophys. J.*, **356**, 359.
Holmberg, J. and Flynn, C.: 2000, *Mon. Not. R. Astron. Soc.*, **313**, 209.
Jaffe, W.: 1983, *Mon. Not. R. Astron. Soc.*, **202**, 995.
Kuijken, K, and Gilmore, G.: 1991, *Astrophys. J.*, **367**, L9.
Navarro, J.F., Frenk, C.S. and White, S.D.M.: 1996, *Astrophys. J.*, **462**, 563.
Navarro, J.F., Frenk, C.S. and White, S.D.M.: 1997, *Astrophys. J.*, **490**, 493.
Nikiforov, I.I., Petrovskaya, I.V. Ninković, S.: 1999, in *"Small Galaxy Groups", ASP Conference Series*, Vol. **3**, eds. M. Valtonen and C. Flynn, p. 1.
Ninković, S.: 1987, *Publ. Astron. Inst. Czech. Acad. Sci.*, No. **69**, p. 317.
Ninković, S.: 1992, *Astroph. Sp. Sci.*, **187**, 159.
Sackett, P.D.: 1997, *Astrophys. J.*, **483**, 103.
Salucci, P.: 2001, *Mon. Not. R. Astron. Soc.*, **320**, L1.
Samurović, S., Ćirković, M.M. and Milošević-Zdjelar, V.: 1999, *Mon. Not. R. Astron. Soc.*, **309**, 63.
Tremaine, S., Richstone, D.O., Byun, Y.-I., Dressler, A., Faber, S.M., Grillmair, C., Kormendy, J. and Lauer, T.R.: 1994, *Astron. J.*, **107**, 634.
Wilkinson, M.I. and Evans, N.W.: 1999, *Mon. Not. R. Astron. Soc.*, **310**, 645.

О ГУСТИНИ У СРЕДИШЊЕМ ДЕЛУ ПОДСИСТЕМА ТАМНЕ МАТЕРИЈЕ У НАШОЈ ГАЛАКСИЈИ

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Оригинални научни рад

Под претпоставком да за диск наше Галаксије, Млечног пута, важи класични експоненцијални закон, да је он максималан и да је Сунце знатно удаљено од места максимума његове кружне брзине добија се да галактичка корона (подсистем који садржи тамну материју), по свој прилици, има скоро константну густину у свом средишњем делу унутар кога је смештено Сунце. Прилаз примењен у овом раду је локалан, тј. обрађују се величине које карактеришу околину Сунца. Полазне претпоставке и резултат рада би могле да објасне зашто се очекује да однос апсолутних вредности Ортових констаната буде већи од вредности 1,0 која одговара случају константне брзине галактичке ротације у околини Сунца.