Serb. Astron. J. № 181 (2010), 57 - 61 UDC 523.31–852
DOI: 10.2298/SAJ1081057S Preliminary report

POSSIBILITIES OF IMPROVING THE TD88 ATMOSPHERIC TOTAL DENSITY MODEL

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(Received: July 7, 2010; Accepted: September 29, 2010)

SUMMARY: In this paper we have examined possibilities for preserving and improving the total density model of the Earth's neutral thermosphere TD88 (Sehnal and Pospíšilová 1988) via modelling differences between TD88 and NRLMSISE-00 (Picone et al. 2002), which is used as a control model. It is shown that these residuals can be approximated with polyharmonic function. Starting from this we have developed the mathematical model of the residuals to identify their origin and possibilities to improve the TD88 model itself.

Key words. Earth

1. INTRODUCTION

Orbital elements of the low Earth satellites ($h < 2000$ km) are strongly influenced by the atmospheric drag. For instance, the International Space Station decays 60m per day on average and a large part of this is due to the atmospheric drag.

This fact allows us to use the data of orbital perturbations for studying the upper Earth atmosphere, and also as a powerful aerodynamic cosmic laboratory for studying characteristics of free molecular, hyperthermal flow which is very hard and expensive to simulate on the Earth.

This analysis can be done mainly by numerical integration of equations of motion of artificial satellites and by comparison of calculated perturbations with measured values. To take into account the effect of atmospheric drag, we need a suitable atmospheric model that can be included in the equations of motion.

Atmospheric models can be generally divided into three groups:

- Empirical,
- Semi-empirical,
- Analytical.

The empirical models are completely experimental and they describe physical parameters of the atmosphere obtained by different kinds of measurements. The semi-empirical models combine analytical formulas and experimental databases to describe the atmospheric parameters, while the analytical models are completely derived from equations that describe physical processes in the atmosphere under assumptions that it is possible to simplify these equations and thus the solutions. The empirical and semi-empirical models are very accurate while the analytical models are less accurate, but have a relatively simple mathematical form suitable for connecting them with the orbital perturbation parameters. These are the reasons for continual development of analytical models based on very accurate empirical and semi-empirical models.

There are several improvements of the TD88 model (Šurlan and Šegan 2009) and they are all based on mathematical upgrading of coefficients of the model without considering physical causes of the errors. Our objective here is to determine whether there are some physical effects that are neglected in the TD88 model and to consider if they can be built into the model to improve its accuracy. As a measure of success we will use estimated standard deviations before and after improving the model.

2. THE TOTAL DENSITY TD88 MODEL

The TD88 model is an improved TD model (Sehnal and Pospíšilová 1988, Bezděk and Vokrouhlický 2004). The air density (ρ) is described by the expression

$$\rho = f_x f_0 k_0 \sum_{n=1}^{7} h_n g_n \ , \qquad (1)$$

where

$f_x = 1 + a_1(F_x - F_b)$,
$f_0 = a_2 + f_m$,
$f_m = (F_b - 60)/160$,
$k_0 = 1 + a_3(K_p - 3)$.

To include the solar and geomagnetic effects and, moreover, individual terms containing factors dependent of the mean solar flux, the terms g_n are used and they are given by the following expressions:

$g_1 = 1$,
$g_2 = f_m/2 + a_4$,
$g_3 = \sin(d - p_3) \sin\varphi$,
$g_4 = (a_5 f_m + 1) \sin(d - p_4)$,
$g_5 = (a_6 f_m + 1) \sin 2(d - p_5)$,
$g_6 = (a_7 f_m + 1) \sin(t - p_6) \cos\varphi$,
$g_7 = (a_8 f_m + 1) \sin 2(t - p_7) \cos^2\varphi$.

By these coefficients the average density, the individual mean solar flux dependence, north-south asymmetry, annual and semi-annual, diurnal and semi-diurnal variations are covered. The height dependence is described by using h_n terms,

$$h_n = K_{n,0} + \sum_{j=1}^{3} K_{n,j} \exp\left(\frac{120 - h}{29j}\right). \qquad (2)$$

The symbols used are: $K_{n,j}$, a_i – numerical constants of the model, p_n – phases, F_x – solar flux measured at 10.7 cm for previous day, F_b – mean solar flux averaged over three solar rotations, K_p – geomagnetic index 3 hours before the current local time, h – altitude, φ – latitude, and t – local time.

3. NRLMSISE-00 THERMOSPHERIC MODEL

The NRLMSISE-00 (US **N**aval **R**esearch **L**aboratory **M**ass **S**pectrometer and **I**ncoherent **S**catter Radar **E**xtended) is an empirical atmospheric model that extends from the ground to the exobase. It is the upgrade of the MSISE-90 model. This model and the associated NRLMSIS database include the following data:

1. Total mass density from satellite accelerometers and from orbit determination (including the Jacchia and Barlier data sets),
2. Temperature from incoherent scatter radar covering the period from 1981 to 1997,
3. Molecular oxygen number density (O_2), from solar ultraviolet occultation aboard the Solar Maximum Mission.

The model depends upon user-provided values:

- Day,
- Time (UT),
- Altitude,
- Latitude,
- Longitude,
- Local solar time,
- Magnetic index (A_p),
- 10.7 cm solar radiation flux index.

The outputs of the model are:

- Number densities of atmospheric constituents (He, O, O_2, N, N_2, Ar, H and anomalous oxygen),
- Total air density,
- Exospheric temperature,
- Temperature at given altitude.

Since this model is based on a large amount of very precise experimental measurements, it is an excellent base for comparison and upgrading the TD88 model.

4. COMPARISON OF THE MODELS

Comparing NRLMSISE-00 and TD88 we noticed that differences in total densities between these two models have periodical character as it is shown in Fig. 1.

By comparison of these models for certain latitude and fixed physical parameters we found that a poly-harmonic approximation is very suitable (representative) for them

$$\rho_{\mathrm{model}} = A + \sum_{i=1}^{n} C_i \cos\left(\frac{2\pi}{T_i} t + \varphi_i\right) \qquad (3)$$

where ρ_{model} is the total density obtained by NRLMSISE-00 or TD88, A is a constant (free term), $C_i s$ are amplitudes of the harmonics which can be modulated, $\phi_i s$ are their phases and $T_i s$ are their assumed periods.

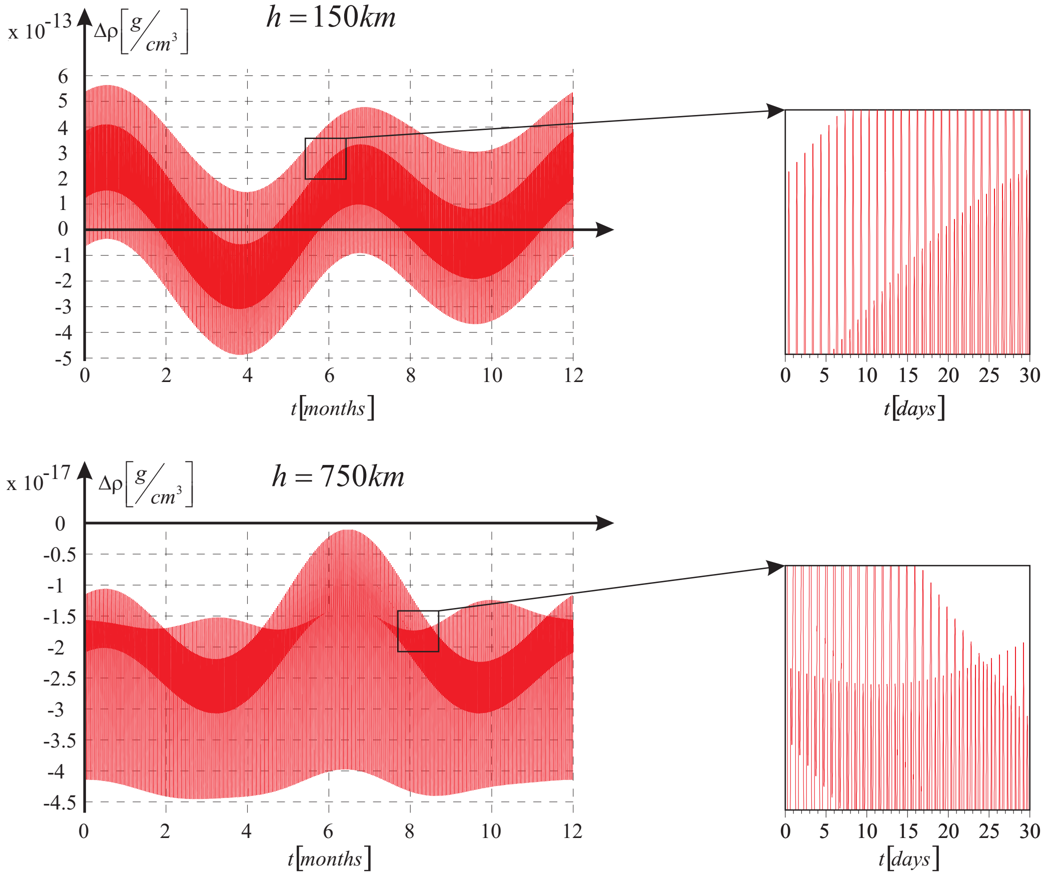

Fig. 1. *Differences in total densities obtained with NRLMSISE-00 and TD88.*

To determine the free term, amplitudes and phases of the harmonics in a least squares sense, it is necessary to linearize this equation with respect to $C_i s$ and $\phi_i s$. Then,

$$\begin{gathered} C_i \cos\left(\frac{2\pi}{T_i}t + \varphi_i\right) = \\ C_i \sin\left(\frac{2\pi}{T_i}\right)\cos\varphi_i + C_i \cos\left(\frac{2\pi}{T_i}t\right)\sin\varphi_i = \\ B_{i1}\sin\left(\frac{2\pi}{T_i}t\right) + B_{i2}\cos\left(\frac{2\pi}{T_i}t\right) \end{gathered} \tag{4}$$

where:

$$\begin{aligned} B_{i1} &= C_i \cos\varphi, \\ B_{i2} &= C_i \sin\varphi. \end{aligned} \tag{5}$$

Calculation of the amplitudes and phases is straightforward now:

$$\begin{aligned} \varphi_i &= \arctan\frac{B_{i2}}{B_{i1}}, \\ C_i &= \sqrt{B_{i1}^2 + B_{i2}^2}. \end{aligned} \tag{6}$$

By substraction of two polyharmonic approximations we get the equation of total densities differences between models in the form

$$\begin{aligned} \Delta\rho = A + \sum_{i=1}^{6} C_i \cos\left(\frac{2\pi t}{T_i} + \varphi_i\right) + \\ \sum_{j=1}^{4} C^j \cos\left(\frac{2\pi t}{T^j} + \varphi^j\right) \end{aligned} \tag{7}$$

where $C^j s$ are modulated amplitudes of short-periodic harmonics in the form

$$C^j = A^j + \sum_{k=1}^{4} C_k^j \cos\left(\frac{2\pi t}{T_k} + \varphi_k^j\right). \tag{8}$$

In Eqs. (7) and (8), free terms, amplitudes and phases are functions of height. The periods of the harmonics are given in the following three tables where [h] and [y] denote hour and year respectively.

Table 1. Periods of unmodulated harmonics

i	1	2	3	4	5
[Dim]	$[h]$	$[h]$	$[y]$	$[y]$	$[y]$
T_i	12	24	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

Table 2. Periods of modulated harmonics

j	1	2	3	4
[Dim]	$[h]$	$[h]$	$[h]$	$[h]$
T_j	6	8	12	24

Table 3. Periods of modulating harmonics

k	1	2	3	4
[Dim]	$[y]$	$[y]$	$[y]$	$[y]$
T_k	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1

Typical dependance of amplitudes and phases on height is shown in Fig. 2.

The standard deviation (σ) of the TD88 model is given by

$$\sigma = \frac{1}{N}\sum_{k=1}^{N}\sigma_k = \frac{1}{N}\sum_{k=1}^{N}\sqrt{(\rho_{0k}-\rho_{1k})^2} \qquad (9)$$

where ρ_{0k} is the total density obtained from NRLMSISE-00, ρ_{1k} is the total density obtained from TD88 model, either before or after adding the modelled residuals (7) and $N = 87600$ is the number of control points (every 6 minutes throughout a year).

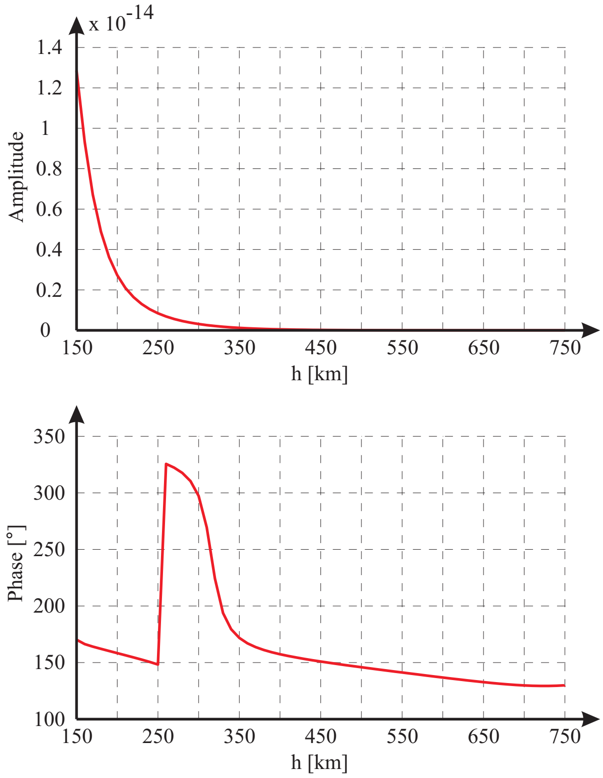

Fig. 2. *Typical dependance of amplitudes and phases on height.*

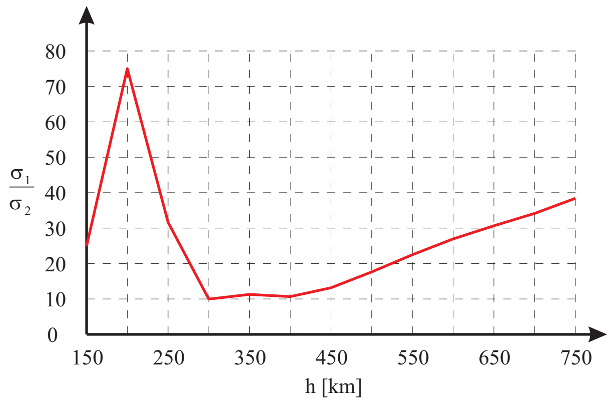

Fig. 3. *The ratio between standard deviations before and after adding of residuals model to the TD88.*

Since Fig. 3 shows that the modelling of residuals lowers the standard deviation of the TD88 at least by one order of magnitude, Eq. (7) allows for comparing models and finding physical origins of the differences between them, and, finally, improvement of the TD88 model.

The free term in the equation shows that TD88 doesn't correctly treat the variation of the total density with height or, in other words, the scale height is not accurate enough.

The next four terms show that amplitudes and phases of semidiurnal, diurnal, semiannual and annual variations of atmospheric density are not defined adequately.

Other terms show that amplitudes of short-periodical harmonics are modulated with long-periodical harmonics and that modulation is not taken into account in the TD88 model.

5. CONCLUSIONS

The above analysis indicates that the earlier methods of improving, and thus preserving of the TD88 model were not adequately chosen, and that it is necessary to take into account our procedure to find an appropriate solution. In that sense, our analysis of the residuals of the TD88 from Eq. (7) shows five problems that should be addressed and possibly solved:

1. Inadequate scale heights,
2. Errors in amplitudes and phases of semidiurnal, diurnal, semiannual and annual component of the atmospheric density,
3. Absence of short-periodic harmonics (6 and 8 hours periods),
4. Absence of intermediary harmonics (3 and 4 months periods),
5. Absence of amplitude modulation of short-periodic harmonics.

By taking into account these five aspects, the TD88 model can be improved to a satisfactory accuracy within a greater interval of heights and without increase in mathematical complexity.

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МОГУЋНОСТИ УНАПРЕЂЕЊА МОДЕЛА ТОТАЛНЕ ГУСТИНЕ АТМОСФЕРЕ TD88

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УДК 523.31–852
Претходно саопштење

У раду смо размотрили могућности за очување и побољшање модела тоталне густине атмосфере (неутралне термосфере) TD88 (Sehnal and Pospíšilová 1988). Моделовали смо резидуе модела TD88 и NRLMSISE-00 (Picone et al. 2002), (контролни модел) и уочили да се они могу врло добро апроксимирати полихармонијском функцијом. Ова чињеница нам омоућује да искористимо математички модел резидуа, да утврдимо њихово порекло и нађемо могућност побољшања модела TD88.