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CORRELATIONS OF ORBITAL ELEMENTS FOR VISUAL DOUBLE STARS

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SUMMARY: In this paper, the authors examine the dependence of correlation coefficients of orbital elements on the length of the orbital arc covered by measurements, on measurements of different accuracies, and on the number of measurements. The obtained correlation coefficients for the orbital elements are found to decrease with the orbital arc length covered by measurements, they are independent of the measurement precision, and they do not depend on the number of measurements for long arcs and they decrease with the number of measurements for short arcs.

Key words. binaries: visual – Methods: data analysis

1. INTRODUCTION

The observations of visual binaries have been collected for more than 200 years, starting from wide pairs resolvable by a naked eye towards more close binaries that can be resolved by various observable techniques. Today, speckle interferometry makes possible measurements of very close binaries with separations less than 0.1 mas.

Many orbits in the Sixth Catalog of Orbits of Visual Binary Stars[1] were determined either from old measurements of low precision or from measurements covering a short orbital arc. All orbits are graded on a 1-5 scale: 1 = definitive, 2 = good, 3 = reliable, 4 = preliminary and 5 = indeterminate. Only about 3% of orbits have the best grade 1. The remaining orbits have lower grades and they need improvement. More recent and more precise measurements offer the possibility to improve already calculated orbits.

The determination of orbital elements of visual binaries is often based on the least-squares method (see Jefferys 1980, 1981, 1988, Eichhorn 1989, Eichhorn and Xu 1990, Pourbaix 1994). These authors calculated the covariance matrices of the system parameter sets, their efficiencies given as measures of the amount of correlation between parameter estimates, and the correlation matrices of the original system parameter sets.

Eichhorn and Xu (1990) pointed out two major difficulties in obtaining estimates of system parameters. First, the conditional equations connecting the measured quantities to the orbital elements

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[1] http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/orb6

are nonlinear and, additionally, one must solve Kepler's transcendental equation too. Second, the measurements cover a short orbital arc with insufficient curvature which makes finding a good orbital solution difficult. For this reason, the system parameters are highly correlated for most binaries.

The objective of this paper is to examine how the correlations for orbital elements depend on the length of the orbital arc covered by measurements, on measurements with different accuracies, and on the number of measurements. These criteria were applied in the selection of binaries. Our sample contains seven orbital pairs whose measurements are taken from the Fourth Catalog of Interferometric Measurements of Binary Stars INT4[2] and from the available database of the Astronomical Observatory in Belgrade. The basic data on these pairs can be found in the Washington Double Star Catalog WDS[3].

2. THE SELECTED SAMPLE OF BINARIES

1. WDS 00155−1608 (HEI 299) is a binary discovered in 1985. There are 24 measurements obtained till 2004 (6 micrometric and 18 interferometric). The first interferometric measurement took place in 1993. For this pair we do not have any older micrometric measurements at our disposal, but only interferometric ones taken from INT4. The measurements cover the full orbit.

2. WDS 00247−2653 (LEI 1) is a three-component system: A, B and C; the measurements used here concern the pair BC. Its measurements started in 1993. There are 44 interferometric measurements made so far. The length of the orbital arc covered by the measurements is 250°.

3. WDS 00321+6715 (MCY 1) is a triple system. The wider pair AB was discovered in 1923. That the A component is a double was discovered only in 1989. For this closer pair (Aa, Ab) that we consider in this paper, there exist interferometric measurements only. By 2005, there are all together 16 of them and they are distributed along the full orbit.

4. WDS 01350−2955 (DAW 31) is a quadruple system. The pair AB we consider here is the closest one in this system and was discovered in 1920. There are 13 of its measurements done by Finsen with his visual interferometer (Finsen 1951), 13 speckle-interferometric measurements, and 69 measurements were performed with the micrometer. The measurements cover the full orbit.

5. WDS 04041+3931 (STF 483) is a binary discovered as early as in 1830. This is a relatively wide pair for which almost all measurements are micrometric and their total number is 133 till 2006. The length of the orbital arc covered by measurements is 315°.

6. WDS 04184+2135 (MCA 14) is a triple system. The wider pair AB was discovered in 1893. That the A component is double was discovered only in 1975. We consider this closer pair (Aa, Ab) for which only speckle-interferometric measurements exist. There are 78 measurements by 2005 covering the full orbit.

7. WDS 04400+5328 (STF 566) has four components: A, B, C and D and, in this paper, we use measurements for the pair AB-C. Within the period 1828-2007, there are 153 measurements and almost all of them are micrometric. The measurements cover the orbital arc of 133°.

3. DESCRIPTION OF METHODS

In our calculations of orbital elements, the Koval'skij-Olević method (Olević and Cvetković 2004) is applied. All observations are assigned the corresponding weights following the proposal given in Hartkopf et al. (1989), Hartkopf et al. (2001).

A general form of an ellipse on the celestial sphere (the apparent orbit of a binary) is

$$a_1x^2 + a_2y^2 + 2a_3xy + 2a_4x + 2a_5y - 1 = 0 \qquad (1)$$

where the vector of parameters $\mathbf{x}=[a_1, a_2, a_3, a_4, a_5]^T$ may be estimated by using the least square method. The standard error of the ith parameter is given by $(K_{ii})^{1/2}$ where $\mathbf{K}$ is the covariance matrix of the vector $\mathbf{x}$.

The errors of orbital elements are calculated in two ways:

(i) Indirectly from errors of five parameters of the apparent orbit $\mathbf{x}=[a_1, a_2, a_3, a_4, a_5]^T$ which are obtained from the covariance matrix of parameters by applying the total-differential formula; their designation is σ.

(ii) Directly from the covariance matrix of orbital elements $\mathbf{J}^{-1} = \mathbf{K}_\alpha$ calculated in this paper; their designation is σ_k. The normal-equation matrix $\mathbf{J}$ (Pourbaix 1994) is formed by using the calculated orbital elements and it is defined as follows:

$$\mathbf{J} = \sum_{i=1}^{n} (\partial(x_i, y_i)/\partial\alpha)^T (\partial(x_i, y_i)/\partial\alpha)/s_0^2 \qquad (2)$$

where α is the vector of orbital elements $\alpha=[P, T, a, e, i, \omega, \Omega]^T$, $s_0^2 = \sum_{i=1}^{n} s_i^2/(n-u)$, n is the total number of observations, u is the number of independent parameters, and the sum of squares of the $(O-C)$ differences is given by $s_i^2 = (x_{o,i} - x_{c,i})^2 + (y_{o,i} - y_{c,i})^2$.

The correlation coefficients between the orbital elements, $C_{ij} = K_{ij}/(K_{ii}K_{jj})^{1/2}$, are calculated from their covariance matrix $\mathbf{K}_\alpha$ and then the correlation matrices of orbital elements are formed in order to examine how the correlation of orbital elements depends on the length of the orbital arc covered by measurements, on the measurements having different accuracy, and on the number of measurements.

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[2] http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/int4

[3] http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/WDS

The sensitivity of the solution for a system of linear equations is studied by the condition number

$$\mathrm{cond}(\mathbf{J}) = \lambda_1/\lambda_n \tag{3}$$

according to Perović (2005), pp. 38-40.

4. RESULTS

The measurements concerning WDS 00247$-$2653 cover an orbital arc of 250° with a gap of about 60° (Fig. 3). As we examine how the correlations of orbital elements depend on the length of the orbital arc covered by measurements, the orbital elements are calculated from both the shorter length arc (the section preceding the gap), and the longer length arc (the entire arc). A similar procedure is applied in the case of WDS 04184+2135 where the measurements cover the whole orbit (Fig. 10). We have a total of 78 measurements out of which 76 are from two orbital periods and the remaining two from the third one. In the case of this pair, it is possible to separate measurements on orbital arcs of different lengths, to use them in calculations of orbital elements, and to examine their correlations.

The orbital elements (equinox J2000) are listed in Table 1. The first column gives the WDS star number, the number of measurements n and the length of orbital arc l. Columns 2-8 contain the orbital elements: P (period, in years), T (epoch of periastron passage, in fractional Besselian year), a (semi-major axis, in arcseconds), e (eccentricity), i (inclination in degrees), ω (longitude of periastron in degrees) and Ω (longitude of node, equinox 2000, in degrees) with corresponding errors σ (calculated indirectly) and σ_k (calculated directly).

For each orbital solution we plot the apparent orbit (Figs. 1-11). In all figures, the micrometric measurements are indicated by the plus sign, the measurements obtained by using visual interferometer (Finsen 1951) are indicated by empty circles, whereas the speckle interferometric ones are given by filled circles. The straight line is the nodal line. The measurements are connected with the corresponding ephemerides. The position of the primary star is indicated by a big plus sign. The arrows in the lower right corner indicate the sense of the companion's revolution around the primary.

The correlation coefficients for orbital elements for all orbital solutions are given in Table 2.

4.1. Discussion

WDS 00155–1608: In Table 1, we can see a good agreement between the errors of orbital elements obtained indirectly (σ) and directly (σ_k). The small values for these errors indicate that the obtained orbital elements are reliable. The apparent orbit is presented in Fig. 1. It offers a good fit to the observed values which confirms that they have been obtained with a high precision. In Table 2, we present the correlation matrix of the orbital elements. Practically all correlation coefficients have small values, or, in other words, its mutual influence is not noticeable. The condition number for the **J** matrix is 9.34×10^2.

WDS 00247–2653: Out of 44 interferometric measurements, 26 of them cover an arc of 126° and they are of high precision as can be seen from Fig. 2. This figure shows the apparent orbit and a high coincidence of the observed and ephemeridal values. The errors σ_k (Table 1) are significantly larger than σ. In this case only one third of the orbit is covered by measurements and the geometry of this arc defines the coefficient matrix for the linear model. Almost all values for the correlation coefficients are close to unity indicating a high correlation of the orbital elements. The condition number of the **J** matrix is 5.95×10^6.

WDS 00247–2653: Now we take all 44 interferometric measurements. The agreement between the errors σ_k and σ is much better (a better geometry) than in the case of the shorter arc. The apparent orbit is presented in Fig. 3. The values for the correlation coefficients are now much smaller. Only three coefficients have absolute values exceeding 0.6. This means that a better geometry of arc reduces the mutual correlation between orbital elements. The condition number of the **J** matrix is significantly lower than in the case of the shorter arc and its value is 3.30×10^4.

WDS 00321+6715: In the case of this pair there are 16 interferometric measurements only. The agreement between the errors σ_k and σ is very good. The apparent orbit is presented in Fig. 4. It yields a good fit to the observed values. The correlation coefficients concerning the orbital elements have small values, except for those between a and i, e and i, P and ω, T and ω, where they slightly exceed 0.5. The condition number of the **J** matrix is 5.96×10^2. It is evident that this value gets smaller if the orbital arc increases. In other words, as the geometry of orbital arc is improved, the orbital elements become more reliable.

WDS 01350–2955: Almost all of the existing measurements are micrometric, and performed by small telescopes of apertures under 0.5 m and of low accuracy, whereas the most recent measurements are speckle-interferometric. The first measurement took place at 1920.86 and the last one at 2008.7673. During this time interval, the secondary made several revolutions around the primary. The errors σ and σ_k are in a very good agreement, except for those concerning ω i Ω. The apparent orbit is presented in Fig. 5 showing higher residuals of the measured values from the ephemerides. The correlation coefficients concerning the orbital elements have low values and only two of them exceed 0.6. The condition number of the **J** matrix is 7.00×10^4 which is by two orders of magnitude bigger than in the case of WDS 00321+6715 (for both pairs the arc is 360°), but the accuracy of measurements is much lower here.

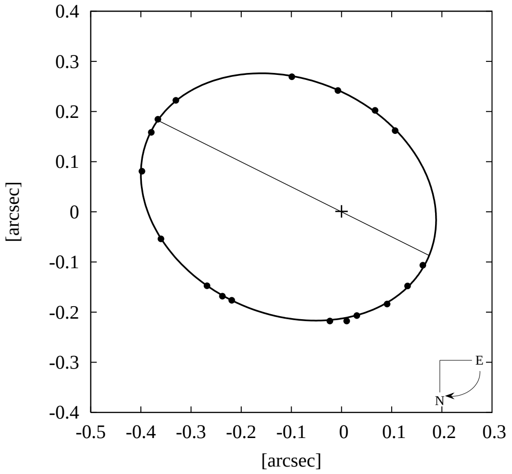

Fig. 1. WDS 00155–1608, *arc* 360°.

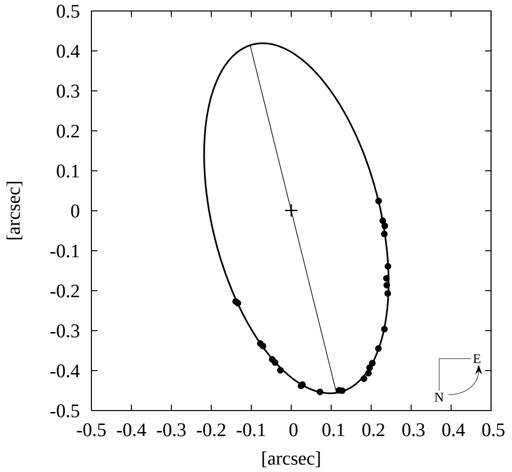

Fig. 2. WDS 00247–2653, *arc* 126°.

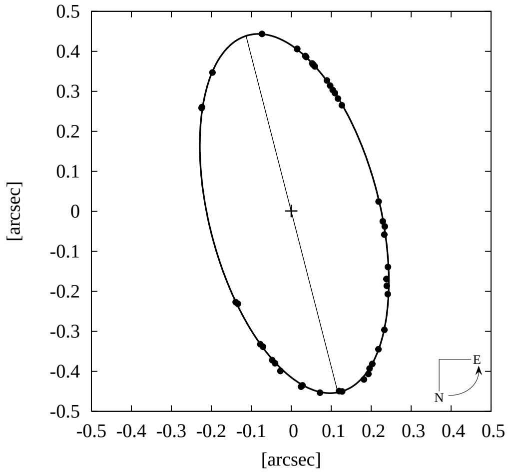

Fig. 3. WDS 00247–2653, *arc* 250°.

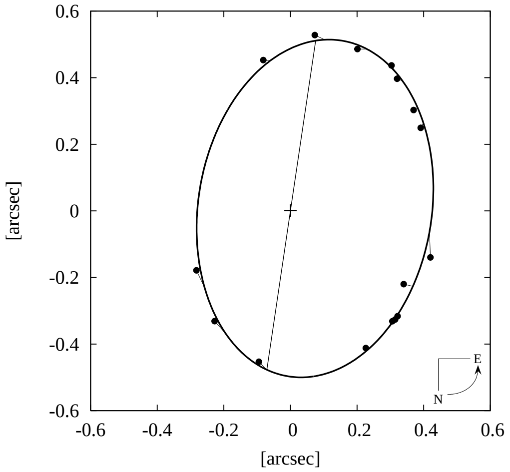

Fig. 4. WDS 00321+6715, *arc* 360°.

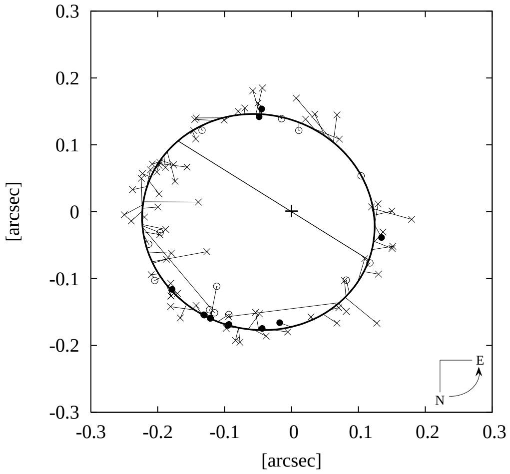

Fig. 5. WDS 01350–2955, *arc* 360°.

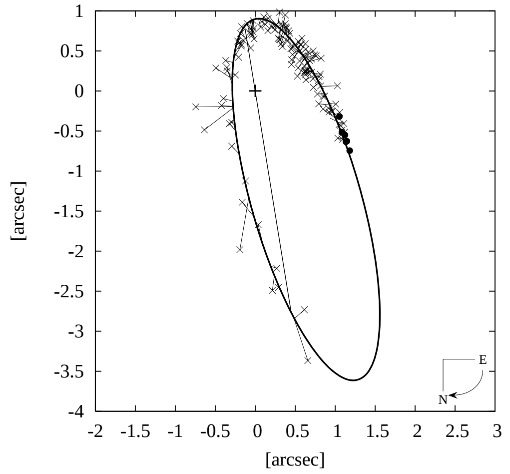

Fig. 6. WDS 04041+3931, *arc* 315°.

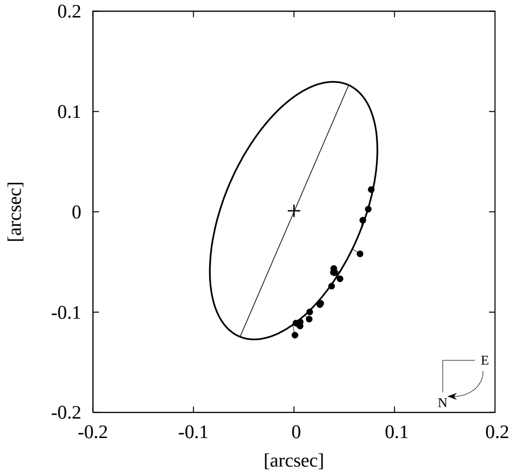

Fig. 7. WDS 04184+2135, *arc* 103°.

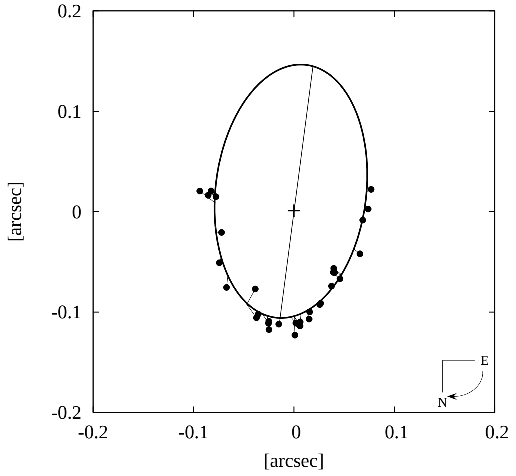

Fig. 8. WDS 04184+2135, *arc* 207°.

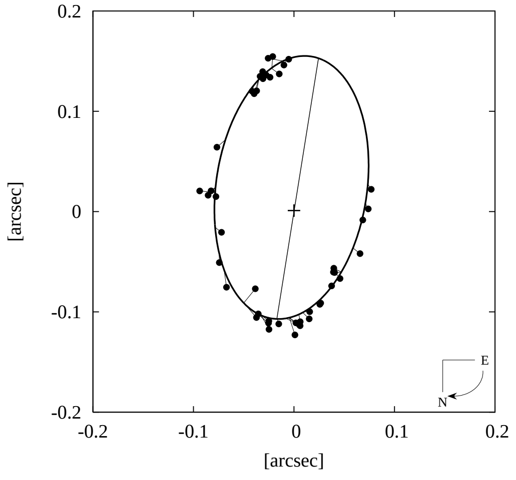

Fig. 9. WDS 04184+2135, *arc* 284°.

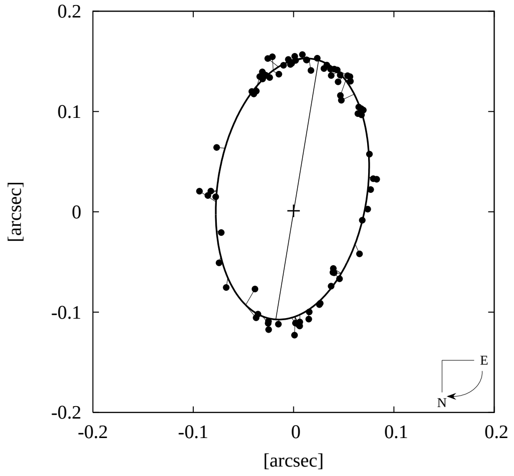

Fig. 10. WDS 04184+2135, *arc* 360°.

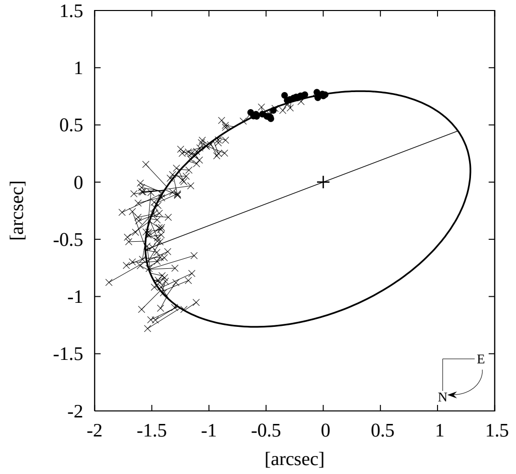

Fig. 11. WDS 04400+5328, *arc* 133°.

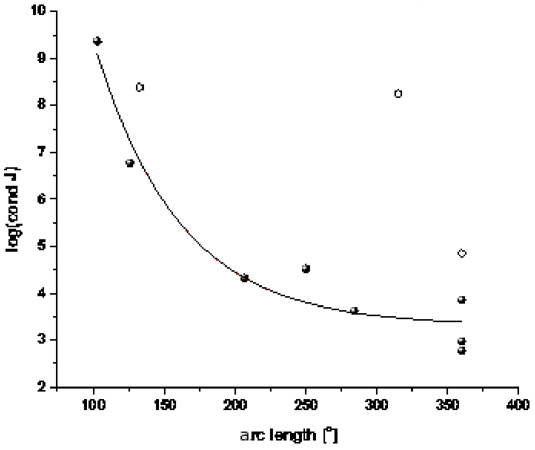

Fig. 12. *Dependence of condition number of matrix of normal equations* **J** *on arc length. The interferometric and micrometric measurements are represented by filled and empty circles respectively.*

WDS 04041+3931: Except for six interferometric measurements, all others are micrometric. This is a long-period binary having a high orbital eccentricity. The errors σ and σ_k are in a good agreement which is not the case for P and T. The apparent orbit is presented in Fig. 6 where a high dispersion of measured values is noticeable due to numerous micrometric measurements. Very small number of correlation coefficients concerning the orbital elements exceeds 0.5. The condition number of the **J** matrix is 1.78×10^8. Such a high value indicates that the solutions can be significantly changed when additional measurements are available.

WDS 04184+2135: All measurements for this binary are speckle-interferometric of high precision which can be seen from Fig. 7 where the observed and ephmeridal values almost coincide. Out of all measurements we select only those covering a short arc of 103° on the part of the orbit which is only slightly different from a straight line. Due to such a geometry the computation of **J** matrix is affected and the errors σ_k are significantly larger than the σ ones. Almost all correlation coefficients are large and only six coefficients are insignificant (about 0.1). The matrix **J** condition number is 2.35×10^9. Such a high value is expected because the orbital elements can be significantly changed by increasing the number of measurements and orbital arc.

WDS 04184+2135: Here we select measurements along a longer orbital arc of 207° which includes the section of ellipse with higher curvature as seen in Fig. 8. The errors σ_k and σ are in a very good agreement and they are noticeably lower than the corresponding values concerning the arc of 103°. It can be seen that the correlation coefficients are smaller here than in the case of the shorter arc. The condition number of matrix **J** is 2.11×10^4 which is significantly lower than in the case when the arc was 103°.

WDS 04184+2135: Let the orbital arc covered by measurements be further enlarged. It is seen that both the agreement between σ_k and σ is improved, and their values are lower. The apparent orbit is presented in Fig. 9. Almost all values for the correlation coefficients are under 0.54. The only case of a somewhat higher coefficient (0.85) concerns the correlation between T and ω. The matrix **J** condition number is now 4.16×10^3 and its value is smaller than in the cases of shorter orbital arcs.

WDS 04184+2135: Now, all the existing measurements are taken into account. The agreement between σ_k and σ is very good. The apparent orbit is presented in Fig. 10. The correlation coefficients are low except for that concerning ω and T which still has a value exceeding 0.8. The **J** matrix condition number is 7.11×10^3.

WDS 04400+5328: We see that σ_k are here much higher than σ and they exceed the errors of WDS 04041+3931. These two pairs can be mutually compared as both of them have long periods and approximately equal number of micrometric measurements from the same period. The apparent orbit is presented in Fig. 11. A significant scatter of micrometric measurements is noticeable. More than half of correlation coefficients are with values over 0.5, and a lot of them are near the value of 1.0. The **J** matrix condition number is 2.44×10^8 and its order of magnitude is the same as for WDS 04041+3931.

The results clearly indicate that the correlations of the orbital elements are smaller when orbital arc covered by measurements is longer and the model geometry is stronger. Reversely, a strong mutual influence of the orbital elements is found for short arcs. Our sample can be divided into three groups. The first one concerns short arcs of 103°, 126° and 133° in which cases the correlation coefficients are close to 1.0. Slightly lower values are noticeable for WDS 04400+5328 for arc 133° with micrometric measurements of lower precision but, in this case, the number of measurements is significantly larger (142) than for the other two pairs (18 and 26) for arcs 103° and 126°. The second group includes the cases with longer arcs: 207°, 250° and 284°. Here, the measurements are interferometric and their number is approximately the same in all three cases. The related correlation coefficients of orbital elements are noticeably smaller than in the case of shorter arcs. The third group contains the cases with long arcs: one of 315° and four complete orbits i.e. 360°. Small values for the correlation coefficients indicate that there are no mutual influences between the orbital elements. A somewhat more prominent mutual influence is found for P, a and e in the case of the long-period binary WDS 04041+3931 (the arc is 315°, eccentricity exceeds 0.7). Neither the influence of the number of measurements (quite different in this group) nor the related measurement accuracies (two pairs have micrometric, three pairs have interferometric measurements) are noticeable.

In Fig. 12, we present the dependence of the condition number of matrix **J** of normal-equations system on the length of the orbital arc covered by measurements. The filled circles represent results related only to pairs measured interferometrically i.e. they possess a high precision. The empty circles represent results related to pairs observed over a sufficiently long time interval and measured mostly micrometrically and, consequently, they are less precise. Since the condition numbers are within a very large range of values, the ordinate axis is taken logarithmic. The figure clearly indicates that the condition number of **J** matrix decreases with increasing length of the orbital arc covered by measurements for both types of measurements. In the case of short arcs, small variations of the initial data (new measurements) can result into significant changes of the orbit solution. The strengthening of the model geometry reduces these changes as seen from the corresponding condition numbers. The results show that measurements of lower precision (micrometric) have higher condition numbers than the more precise interferometric measurements. In our sample, the exception is WDS 04041+3931 with less precise micrometric measurements covering the arc of 315° while the condition number is of the same order of magnitude as for the arc of 130°. An inspection of the apparent orbit shows (Fig. 11) that there are few measurements which are non-uniformly distributed and have significant scatter at the beginning of the arc (the part of about 120°). These measurements could significantly affect the determination of orbital elements as well as the obtained condition number. This is not the case with the other long-period pair WDS 04400+5328 where the old micrometric measurements have a larger scatter but they are uniformly distributed along the arc of 133°.

The greater the condition number, the stronger is the correlation between the orbital elements and vice versa. Both quantities are higher for pairs where measurements cover shorter orbital arcs.

As for the accuracy of the determination of orbital elements, the results indicate that a higher accuracy (lower values for σ and σ_k) corresponds to a longer orbital arc covered by measurements, and to more precise measurements. The comparison of error estimates obtained indirectly (σ) and those obtained directly from the covariance matrix of orbital elements (σ_k) indicates larger differences in the case of shorter arcs. No significant difference of these estimates is found in the case of less precise measurements. This means that the model geometry has the strongest influence.

5. CONCLUSION

The correlations of the orbital elements become smaller as the length of the orbital arc covered by measurements increases, i.e. as the model geometry becomes stronger. Reversely, a stronger mutual influence of orbital elements is found in the case of arcs shorter than 180°. In the case of long arcs the correlation is independent of the number of measurements which seems not to be the case for short arcs where the correlation becomes smaller as the number of measurements increases. The correlation does not depend on the measurement precision.

Table 1. Orbital elements and their errors.

WDS	P[god]	T	$a['']$	e	$i[°]$	$\omega[°]$	$\Omega[°]$
n	σ_P	σ_T	σ_a	σ_e	σ_i	σ_ω	σ_Ω
l	σ_{kP}	σ_{kT}	σ_{ka}	σ_{ke}	σ_{ki}	$\sigma_{k\omega}$	$\sigma_{k\Omega}$
00155−1608	4.559	1995.359	0.3046	0.3636	145.58	346.99	63.54
18	0.008	0.009	0.0170	0.0196	0.31	0.63	0.53
360°	0.004	0.005	0.0013	0.0022	0.62	1.26	1.04
00247−2653	16.209	2006.593	0.4487	0.0604	62.08	218.10	13.95
26	0.063	0.026	0.0116	0.0060	0.28	4.53	0.33
126°	1.517	2.534	0.0323	0.0645	1.72	36.44	0.95
00247−2653	17.039	2008.200	0.4606	0.0325	62.40	241.30	14 45
44	0.019	0.012	0.0354	0.0149	0.78	24.13	1.01
250°	0.044	0.147	0.0011	0.0032	0.20	3.27	0.19
00321+6715	15.837	2000.266	0.5215	0.2076	47.71	99.54	171.57
16	0.146	0.123	0.0803	0.0338	1.08	4.96	1.54
360°	0.250	0.232	0.0161	0.0241	3.07	5.27	4.03
01350−2955	4.557	1991.889	0.1787	0.3047	22.60	51.53	58.04
95	0.023	0.023	0.0474	0.0805	2.70	6.27	5.75
360°	0.004	0.054	0.0060	0.0177	6.77	18.07	18.27
04041+3931	503.231	1910.072	2.6307	0.7196	109.50	319.54	189.24
133	5.539	4.146	0.2395	0.0412	0.79	3.65	1.04
315°	35.919	0.791	0.1186	0.0154	0.70	1.70	1.18
04184+2135	11.256	1977.297	0.1366	0.0137	120.37	120.53	156.70
18	0.032	0.037	0.0062	0.0198	1.84	125.59	1.82
103°	0.043	114.502	0.0420	0.2307	21.60	3418.16	33.71
04184+2135	11.287	1977.592	0.1271	0.1724	126.71	154.49	172.56
32	0.022	0.026	0.0044	0.0229	1.53	7.61	2.46
207°	0.054	0.338	0.0042	0.0205	2.25	9.72	4.69
04184+2135	11.241	1977.641	0.1325	0.1941	125.14	155.19	170.93
47	0.061	0.079	0.0044	0.0241	1.47	7.13	2.10
284°	0.046	0.111	0.0027	0.0118	1.84	4.25	2.02
04184+2135	11.326	1977.766	0.1315	0.1811	125.15	159.40	170.62
78	0.050	0.071	0.0034	0.0209	1.12	6.26	1.58
360°	0.033	0.078	0.0013	0.0065	1.01	2.98	1.07
04400+5328	480.754	2027.788	1.5138	0.2289	128.42	307.11	110.80
142	3.806	3.257	0.0377	0.0121	0.61	4.97	0.78
133°	69.618	20.268	0.1477	0.0624	4.55	30.48	4.59

Table 2. Correlations of orbital elements.

	P	T	a	e	i	ω	Ω	P	T	a	e	i	ω	Ω
	WDS 00155−1608			n=18		luk=360°		WDS 04184+2135			n=18		luk=103°	
P	1.00	-0.07	-0.12	0.43	0.17	-0.07	-0.11	1.00	-0.11	-0.08	-0.09	0.09	-0.11	-0.11
T		1.00	-0.24	0.02	0.39	0.61	0.37		1.00	0.97	0.85	-0.95	1.00	0.98
a			1.00	-0.35	-0.66	-0.11	-0.04			1.00	0.69	-0.99	0.96	0.90
e				1.00	0.28	-0.20	-0.24				1.00	-0.65	0.85	0.93
i					1.00	0.19	0.07					1.00	-0.95	-0.88
ω						1.00	0.95						1.00	0.98
Ω							1.00							1.00
	WDS 00247−2653			n=26		luk=126°		WDS 04184+2135			n=32		luk=207°	
P	1.00	0.98	1.00	-0.99	0.97	0.95	0.59	1.00	-0.09	0.04	0.04	-0.01	-0.04	-0.08
T		1.00	0.99	-0.95	0.98	0.99	0.42		1.00	0.15	-0.70	-0.06	0.97	-0.82
a			1.00	-0.99	0.99	0.96	0.55			1.00	0.35	-0.66	0.13	-0.09
e				1.00	-0.96	-0.91	-0.67				1.00	-0.28	-0.67	0.66
i					1.00	0.97	0.49					1.00	-0.07	-0.03
ω						1.00	0.32						1.00	-0.67
Ω							1.00							1.00
	WDS 00247−2653			n=44		luk=250°		WDS 04184+2135			n=47		luk=284°	
P	1.00	-0.38	-0.29	-0.84	-0.49	-0.50	0.26	1.00	-0.14	0.16	0.09	-0.07	0.03	-0.15
T		1.00	0.39	0.59	0.50	0.99	-0.09		1.00	-0.20	-0.51	0.33	0.85	-0.06
a			1.00	0.32	0.53	0.40	0.01			1.00	0.27	-0.51	-0.37	-0.27
e				1.00	0.59	0.69	-0.33				1.00	-0.32	-0.54	-0.05
i					1.00	0.54	-0.26					1.00	0.30	-0.22
ω						1.00	-0.17						1.00	0.35
Ω							1.00							1.00
	WDS 00321+6715			n=16		luk=360°		WDS 04184+2135			n=78		luk=360°	
P	1.00	-0.32	0.15	0.30	0.22	-0.53	0.05	1.00	-0.09	0.00	-0.02	-0.01	0.16	-0.05
T		1.00	-0.04	-0.04	-0.06	0.65	0.34		1.00	-0.07	-0.30	-0.02	0.88	0.33
a			1.00	0.29	0.64	-0.16	0.10			1.00	-0.34	-0.41	-0.11	-0.08
e				1.00	0.57	-0.34	0.28				1.00	0.21	-0.27	-0.15
i					1.00	-0.23	0.14					1.00	-0.02	-0.08
ω						1.00	-0.42						1.00	0.63
Ω							1.00							1.00
	WDS 01350−2955			n=95		luk=360°		WDS 04400+5328			n=142		luk=133°	
P	1.00	0.60	0.05	-0.11	0.05	0.15	-0.15	1.00	-0.96	0.93	-0.10	-0.40	-0.98	-0.75
T		1.00	0.28	-0.03	0.37	0.13	0.00		1.00	-0.87	-0.05	0.28	0.99	0.88
a			1.00	-0.08	0.73	0.11	-0.06			1.00	-0.43	-0.69	-0.93	-0.58
e				1.00	0.09	0.44	-0.42				1.00	-0.91	0.11	-0.38
i					1.00	0.06	0.02					1.00	0.42	-0.06
ω						1.00	-0.98						1.00	0.82
Ω							1.00							1.00
	WDS 04041+3931			n=133		luk=315°								
P	1.00	0.21	0.91	0.88	-0.15	0.45	0.12							
T		1.00	0.08	-0.11	0.47	0.74	-0.03							
a			1.00	0.85	-0.40	0.32	0.11							
e				1.00	-0.35	0.02	0.05							
i					1.00	0.36	0.01							
ω						1.00	0.37							
Ω							1.00							

The condition number of matrix **J** becomes smaller as the length of the orbital arc covered by measurements increases for both types of measurements. In the case of short arcs, small changes of the initial data (new measurements) can alter the orbit solution significantly. As the model geometry becomes stronger, the changes of the orbit solution become smaller and smaller. The measurements of lower precision (micrometric) have higher condition numbers than the more precise interferometric measurements.

The condition number and correlation between the orbital elements are larger for pairs where the measurements cover shorter orbital arcs and vice versa.

The accuracy of determination of orbital elements is higher for longer orbital arcs covered by measurements, and for more precise measurements. The error estimates obtained indirectly and directly from the covariance matrix of orbital elements are significantly different for short arcs but not for long arcs.

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КОРЕЛАЦИЈЕ ПУТАЊСКИХ ЕЛЕМЕНАТА ВИЗУЕЛНО ДВОЈНИХ ЗВЕЗДА

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Стручни чланак

У раду је испитивана зависност коефицијената корелације путањских елемената од дужине лука орбите покривеног мерењима, од мерења различите тачности и од броја мерења. Добијени коефицијенти корелације путањских елемената показују 1) смањење са повећањем лука орбите покривеног мерењима; 2) не зависе од тачности мерења и 3) у случају дугачких лукова - не зависе од броја мерења, а у случају кратких лукова - смањују се са порастом броја мерења.