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Addendum

MAXIMUM MASS RATIO OF AM CVn-TYPE BINARY SYSTEMS AND MAXIMUM WHITE DWARF MASS IN ULTRA-COMPACT X-RAY BINARIES

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SUMMARY: We recalculated the maximum white dwarf mass in ultra-compact X-ray binaries obtained in an earlier paper (Arbutina 2011), by taking the effects of super-Eddington accretion rate on the stability of mass transfer into account. It is found that, although the value formally remains the same (under the assumed approximations), for white dwarf masses $M_2 \gtrsim 0.1 M_{\rm Ch}$ mass ratios are extremely low, implying that the result for $M_{\rm max}$ is likely to have little if any practical relevance.

Key words. binaries: close – gravitational waves – stars: white dwarfs

In an earlier paper (Arbutina 2011), by considering the dynamical stability of mass transfer in ultra-compact X-ray binaries (UCXB), we obtained the maximum mass of white dwarf secondary $M_{\rm max} \approx 0.738\ M_{\rm Ch} \approx 1.06\ M_\odot$ and the maximum mass ratio of AM CVn-type binary systems $q_{\rm max} = 0.634$. However, there is an additional restriction for stability, that requires the initial mass transfer rate be below the Eddington limit for the accretor (Nelemans et al. 2001, Marsh et al. 2004). In the case of the super-Eddington accretion the matter that cannot be accreted is lost from the system, along with some angular momentum. Although the binary system may remain stable, the expanding matter will probably form a common envelope in which the two components are most likely to merge (Han and Webbink 1999). We shall assume that the white dwarf's companion is a black hole and see what consequences the super-Eddington accretion has on $M_{\rm max}$.

We assume the presence of the accretion disk around a black hole and start with the condition:

$$L < L_{\rm Edd} \tag{1}$$

where:

$$L = -GM_1\dot{M}_2\Big(\frac{1}{R_I} - \frac{1}{R_{\rm L1}}\Big) \approx -\frac{GM_1\dot{M}_2}{R_I} \tag{2}$$

is the luminosity of the disk, R_I is the radius of the accretor, $R_{\rm L1}$ is the distance of the first Lagrangian point to the centre of the accretor (see Postnov and Yungelson 2006) and:

$$L_{\rm Edd} = \frac{4\pi GM_1 c}{\varkappa} \tag{3}$$

is the Eddington luminosity. Opacity for the Thomson scattering is $\varkappa = \frac{\sigma_e n_e}{\rho}$, $\sigma_e = \frac{8\pi}{3}(\frac{e^2}{4\pi\epsilon_o m_e c^2})^2$ is the Thomson's cross-section, $n_e = \frac{\rho}{2m_p}(1+X)$ is the electron number density, ρ the density and $X \approx 0$ the hydrogen mass fraction. For the "effective surface" of the black hole we shall take the last stable orbit with $R_I = \frac{6GM_1}{c^2}$ (Shapiro and Teukolsky 1983). By combining Eqs. (1), (2) and (3) we find the condition:

$$|\dot{M}_2| < \frac{16\pi c m_p R_I}{\sigma_e}. \tag{4}$$

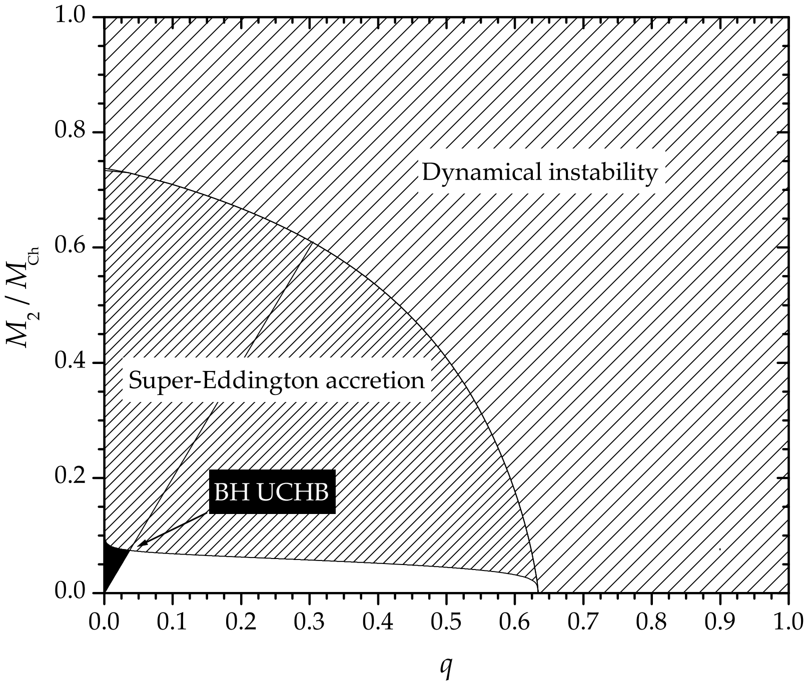

Fig. 1. *The $q_{\rm max} = q_{\rm max}(M_2)$ relation for white dwarfs in UCXB. The line $M_2/M_{\rm OV} = q$ represents the limit above which we have a black hole in an ultra-compact X-ray binary (BH UCXB). In the black area, the accretion rate is dynamically stable and sub-Eddington. Since the derivation assumed a black hole primary, the white area has no physical relevance.*

From Marsh et al. (2004) and Arbutina (2011) we have:

$$\frac{\dot{M}_2}{M_2} = \frac{\dot{J}}{J}\left(\frac{\zeta(M_2)}{2} - \frac{\eta(q)}{6} + 1 - q - \sqrt{(1+q)\frac{R_I}{a}}\right)^{-1}$$

$$\approx \frac{\dot{J}}{J}\Big(\frac{\zeta(M_2)}{2} - \frac{\eta(q)}{6} + 1 - q\Big)^{-1}, \tag{5}$$

where we have neglected the last term $\sqrt{(1+q)R_I/a}$, since $a \gtrsim R_2 \gtrsim 1000$ km and $R_I \sim 10$ km for a solar mass black hole. The angular momentum loss due to gravitation radiation is:

$$\frac{\dot{J}}{J} = -\frac{32}{5}\frac{GM_1M_2M}{c^5a^4}. \tag{6}$$

From Eqs. (4), (5) and (6), assuming $R_2 \approx R_{\rm IL2}$, where $R_{\rm IL2}$ is the inner Roche lobe for the secondary, we obtain after some algebra:

$$\frac{1}{2}\zeta(M_2) - \frac{1}{6}\eta(q) + 1 - q \;>$$

$$k\Big(\frac{M_2}{M_{\rm Ch}}\Big)^3\frac{1+q}{q}\Big(\frac{R_{\rm IL2}}{a}\Big)^4\Big(\frac{R_2}{R_o}\Big)^{-4} \tag{7}$$

where:

$$k = \frac{8}{45}\frac{G^2M_{\rm Ch}^3}{R_o^4m_{\rm p}c^4}\Big(\frac{e^2}{4\pi\epsilon_o m_{\rm e}c^2}\Big)^2, \tag{8}$$

$\frac{M_{\rm Ch}}{M_\odot} = 1.456\Big(\frac{2}{\mu_{\rm e}}\Big)^2$, $\frac{R_o}{R_\odot} = 5.585\cdot 10^{-3}\Big(\frac{2}{\mu_{\rm e}}\Big)$ and $\mu_{\rm e} \approx 2$ (see Hansen and Kawaler 1994).

To find the region of stability in the (M_2, q) plane (see Fig. 1) we have to solve Eq. (7) numerically. Since we are dealing with a black hole primary, we only consider masses $M_1 > M_{\rm OV}$, where, given all the uncertainties, we took for the Oppenheimer-Volkoff mass $M_{\rm OV} \approx 2M_{\rm Ch}$. In the limit $M_2 \to 0$, $R_2 \propto M_2^{-1/3}$, while in the limit $q \to 0$, $R_{\rm IL2}/a \propto q^{1/3}$, so that in both cases the right hand side of Eq. (7) tends to zero, meaning that the values for $M_{\rm max}$ and $q_{\rm max}$ will remain the same as in Arbutina (2011). However, since the constant k is large, the curve $q_{\rm max} = q_{\rm max}(M_2)$ separating the stable and unstable region (after considering the possible super-Eddington accretion) will be shifted to lower white dwarf masses (Fig. 1), except for the extremely low mass ratios, implying that the result for $M_{\rm max}$ is likely to have little if any practical relevance.

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