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KASNER TYPE MAGNETIZED STRING COSMOLOGICAL MODELS IN $f(R,T)$ GRAVITY

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SUMMARY: The Bianchi-I Kasner type metric with cosmic string and magnetic field in the framework of the $f(R,T)$ theory of gravitation is considered. Three different functional forms of the function $f(R,T)$ are chosen for investigation. We found that the strings exist in early stages of evolution of the Universe and they disappear as time increases. The variation of the equation of state (EoS) parameter $\omega = p/\rho < -1$ may come from the effect of the string. We find that the string tension and rest energy density reduce in presence of magnetic field. The Universe is expanding and accelerating.

Key words. Gravitation – Early Universe – Dark matter

1. INTRODUCTION

Early stages of evolution of the Universe is an exciting field of research in recent years. Spontaneous breaking of symmetry of the Universe causes topological defects such as cosmic string, domain walls, monopoles etc. (Kibble et al. 1994). Stress energy of a cosmic string couples to gravitational field in a simple way. The general relativistic study of a cosmic string is initiated by Stachel (1980). It is believed that strings are responsible for density perturbations which leads to formation of galaxies (Zeldovich 1980). Banerjee et al. (1990) have obtained solutions for the Bianchi type I metric in presence of massive string. Bali and Dave (2001) have studied cosmic string in context of the general relativity. Katore and Hatkar (2015) have investigated homogeneous hypersurface with cosmic string in Lyra geometry. Naidu et al. (2013) have obtained solutions for bulk viscous strings in the Brans-Dicke theory using the five dimensional Kaluza-Klein space time. Katore (2015) has described string cosmological models for the Bianchi type II, VIII and IX metric.

Cosmological observations (Riess et al. 1998, Spergen et al. 2003, Tegmark et al. 2004) indicate that the Universe is accelerating and expanding. It is believed that cosmic acceleration is driven by some kind of energy with negative pressure known as dark energy. As the nature of dark energy is yet unknown, modification of the Einstein-Hilbert action of general relativity is proposed. These theories are called modified theories of gravitation. Among them, the $f(R)$ modified gravity theory is widely studied. The $f(R)$ modified gravity theory has succesfully derived the late time acceleration of the Universe (Nojiri and

Odintsov 2007). From the review of the literature, it is found that Capozziello et al. (2008), Multamaki and Vilja (2007), Katore and Hatkar (2016a), Shamir (2010), Chibba et al. (2007) are some of the researchers who have discussed several aspects of the $f(R)$ gravity. Recently, Harko et al. (2011) have proposed another alternative theory of gravitation known as the $f(R,T)$ theory of gravity. In the $f(R,T)$ theory, the arbitrary function $f(R,T)$, where R is the Ricci scalar and T is the trace of the stress-energy tensor T_{ij}, is introduced in the Lagrangian. Houndjo (2012) has discussed the transition from a matter dominated phase to an accelerated phase in the $f(R,T)$ gravity. Exponential solution of the Universe is obtained by Bamba et al. (2012). The domain wall effect for the Bianchi type III and Kantowski-Sachs is studied by Katore and Hatkar (2016b). The case of perfect fluid for the Bianchi type III in $f(R,T)$ is investigated by Reddy et al. (2012). Moraes (2014) has presented unification of the Kaluza-Klein extra-dimensional model with the $f(R,T)$ gravity.

On the large scale, the Universe seems homogeneous and isotropic. But there is no observational data that guarantees the isotropy in an era prior to recombination. The observed local anisotropies in galaxies, cluster and super cluster led idea to explore anisotropic models (Cataldo and Campo 2002). The Kasner solution describes an anisotropic metric. The Kasner metric is invariant under a three dimensional Abelian translation group. Taub has derived Kasner solution which represents an idealized Universe that is expanding in a higher anisotropic manner. The Kasner solution plays an important role in study of anisotropy in quantum particle creation, Baryosynthesis, inflation, massive particle survival, magnetic field evolution, primordial nucleosynthesis, temperature isotropy and statics of the microwave background (Paliathansis et al. 2018). The Kasner solution arises very naturally when one formulates the Einstein field equations for non tilted spatially homogeneous cosmologies as a dynamical system (Wainwright and Krasinsk 2008). The Kasner solution described the evolution of Mixmaster Universe when the effect of the Ricci scalar of the three dimensional spatial hypersurface is negligible because of simplicity and the importance of the Kasner solution. It was studied in higher dimensional and various modified theories. The existence of solution in higher order theories is studied by Clifton (2006). Skugoreva and Toporensky (2018) have investigated the Kasner solution in the $f(T)$ cosmology. In the second order gravity, the Kasner solution is an asymptotic solution whereas in the fourth order gravity it is an exact vacuum solution. The theory in which the Lagrangian is proportional to R^n admitting exact solutions for the Friedmann-Robertsorn-Walker models. It also generalizes the Schwarschild metric. These solutions provide a testing ground for new developments in gravitation theory such as particle production, Holography, gravitational thermodynamics (Clifton and Barrow 2005). Moreover, Clifton and Barrow (2006) have investigated the initial singularity by finding exact cosmological solutions in the fourth order gravity theory with help of the Kasner metric. Gao and Shen (2016) found a new method for static and spherically symmetric solutions in the $f(R)$ theory of gravity. Paliathansis (2016) has obtained new integrable $f(R)$ models from the Killing tensors. Camanhio et al. (2016) have reviewed pure Lovelock equations for the Kasner metrics.

Inspired by the above studies in the field, in the present work we intend to obtain solutions to a magnetized string for three different functional forms of the function $f(R,T)$. The paper is organized as follows: in Section 2, we present metric and field equations. In Section 3, we obtain solutions of the field equations for the functional form $f(R,T) = R + 2f(T)$. In Section 4 and 5, we solve field equations for the functional forms $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = \mu R + \mu T$, respectively. In Section 6, we summarize our results.

2. METRIC AND FIELD EQUATIONS

Friedmann-Robertsom-Walker (FRW) models are the best for representation of the present large scale structure of the Universe. The nature of the FRW model is homogeneous and isotropic. When we think of the early Universe, it is believed that it is not like today i.e. it may not be isotropic. In order to know the early structure of the Universe, we should consider different models. In this sense the Bianchi type models are the simplest, homogeneous and anisotropic and therefore they are important to study the beginning of the Universe. In the Bianchi model, the spatial section is flat in which the extension or contraction rate is direction dependent. Adhav (2012) has solved field equations of the $f(R,T)$ gravity for the Bianchi type I space time. Saaidi et al. (2010) have studied the $f(R)$ modifications of Einstein's gravity in various Bianchi type I for the Kasner form metric. Here, we intend to deal with the LRS Bianchi type I space time of the Kasner type in the following form:

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2, \quad (1)$$

where $p_1,\ p_2$ and p_3 are three parameters satisfying $p_1 + p_2 + p_3 = s,\ p_1^2 + p_2^2 + p_3^2 = \theta$. The parameters $p_1,\ p_2,\ p_3$ will require to be constants and if at least two of the three are different, the space is anisotropic. The energy momentum tensor for a magnetized string is given by:

$$\begin{aligned} T_{ij} &= \rho\mu_i\mu_j - \lambda x_i x_j \\ &+ \frac{1}{4\pi}\left(-g^{\alpha\beta}F_{i\alpha}F_{j\beta} + \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right), \end{aligned} \quad (2)$$

where ρ is the total rest energy density of the fluid and λ is the string tension density. ρ_p is the particle energy density. It satisfies the condition $u^i u_i = -x^i x_i = 1$. In a co-moving co-ordinate system $u^1 = u^2 = u^3 = 0, u^4 = 1$. Recently, Yilmaz (2006) has investigated the problem of string and domain wall for quark matter in the theory of general relativity. This type of case in a scale covariant theory of gravitation is also studied by Reddy and Naidu (2007). Katore et al. (2016a) have obtained solutions for the Bianchi type I space-time with string in presence and absence of magnetic field in context of the $f(R)$ theory of gravitation. Sharma and Sigh (2014) have explored the Bianchi type II space-time for cosmic string and magnetic field in context of the $f(R,T)$ theory of gravitation. We will take the occurance of magnetic filed in direction of the y axis. Therefore, there is only one non-vanishing component, namely $F_{31}=I$. Maxwell's field equations are:

$$F_{ij;k}+F_{jk;i}+F_{ik;j}=0, \tag{3}$$

and:

$$F^{ij}_{;j}=0. \tag{4}$$

We have the following relation between the energy density and trace of the energy momentum tensor:

$$\rho=\rho_p+\lambda, \tag{5}$$

$$T=\lambda+\rho. \tag{6}$$

As discussed above, Harko et al. (2011) have proposed the following action for the $f(R,T)$ gravity:

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x. \tag{7}$$

From this, we can derive field equations of the $f(R,T)$ gravity model discovered by Harko et al. (2011) and we find:

$$\begin{aligned}&f_R(R,T)R_{ij}-\frac{1}{2}(R,T)g_{ij}\\&+(g_{ij}\Box-\nabla_i\nabla_j)f_R(R,T)\\&=8\pi T_{ij}-f_T(R,T)Tij-f_T(R,T)\Theta_{ij},\end{aligned} \tag{8}$$

where:

$$T_{ij}=\frac{-2\partial(\sqrt{-g})}{\sqrt{-g}\partial g^{ij}}L_m,$$

$$\Theta_{ij}=-2T_{ij}-pg_{ij},$$

$$f_R(R,T)=\frac{\partial f(R,T)}{\partial R},$$

$$f_T(R,T)=\frac{\partial f(R,T)}{\partial T},\ \Box=\nabla^{\mu}\nabla_{\mu}\,.$$

Here, one can derive the covariant derivative ∇_μ and matter energy momentum tensor T_{ij} from the Lagrangian L_m. As stated earlier, $f(R,T)$ is a function of R and T. Further L_m is referred to the matter Lagrangian density. In the work of Harko et al. (2011), we see that the three cases of function $f(R,T)=R+2f(T)$, $f(R,T)=f_1(R)+f_2(T)$ and $f(R,T)=f_1(R)+f_2(R)f_3(T)$ are discussed. It should mention that formation of the functional $f(R,T)$ is governed by the matter field. Therefore, different choices of the functional are possible which may lead to different models. We shall now proceed to obtain solutions of the field equations by taking the previous forms of the function $f(R,T)$.

3. MODEL I

We start by assuming the following function discussed by Harko et al. (2011):

$$f(R,T)=R+2f(T), \tag{9}$$

where the function $f(T)$ depends only on the trace of the matter field. This simple functional form is studied by many authors. Katore et al. (2016b) have presented a homogeneous hypersurface metric with perfect fluid in the $f(R,T)$ theory of gravitation. Sahoo et al. (2016) have studied the Bianchi type III and VI_0 cosmological models with a string fluid source in the $f(R,T)$ gravitation theory by considering the function given in Eq. (9). Singh and Singh (2014) have investigated cosmological models by choosing this particular function given in Eq. (9). Pasqua et al. (2013) have found that for $f(R,T)=\mu R+\nu T$ the equation of state parameter approaches -1 and the deceleration parameter transits from the decelerated to accelerated phase at the redshift of $z\approx 0.2$. Using Eqs. (8), (9), the gravitational field equations reduce to:

$$\begin{aligned}G_{ij}&=R_{ij}-\frac{1}{2}Rg_{ij}\\&=8\pi T_{ij}+2f'(T)T_{ij}\\&+[2pf'(T)+f(T)]g_{ij},\end{aligned} \tag{10}$$

where the prime indicates the differentiation with respect to the argument. We also choose the function of matter as:

$$f(T)=\mu T, \tag{11}$$

where μ is constant. In the co-moving coordinate system, from Eqs. (1), (2), (3), (6), (10) and (11), it is easy to write down the expressions for the field equations:

$$\begin{aligned}&[p_1\,(s-1)-K_0]t^{-2}\\&=-\frac{(8\pi+2\mu)\,I^2t^{(2p_1+2p_3)}}{8\pi}\\&+2\mu p+\mu\rho+\mu\lambda,\\&[p_2\,(s-1)-K_0]t^{-2}\\&=\frac{(8\pi+2\mu)\,I^2t^{(2p_1+2p_3)}}{8\pi}\\&+2\mu p+\mu\rho+(8\pi+3\mu)\,\mu\lambda,\end{aligned} \tag{12}$$

$$[p_3\,(s-1)-K_0]t^{-2} = -\frac{(8\pi+2\mu)\,I^2t^{(2p_1+2p_3)}}{8\pi} +2\mu p+\mu\rho+\mu\lambda, \quad (13)$$

$$[\frac{1}{2}\,(\theta-s^2)]t^{-2} = -\frac{(8\pi+2\mu)\,I^2t^{(2p_1+2p_3)}}{8\pi} +2\mu p+(8\pi+3\mu)\,\rho+\mu\lambda, \quad (14)$$

where $K_0 = \frac{1}{2}\left(s^2-2s+\theta\right)$. From Eqs. (12) and (14), we get $p_1 = p_3$. For this value, the above system of Eqs. (12)-(14) reduces to the following equations:

$$[p_1\,(s-1)-K_0]t^{-2} = -\frac{(8\pi+2\mu)\,I^2t^{(4p_1)}}{8\pi} +2\mu p+\mu\rho+\mu\lambda, \quad (15)$$

$$[p_2\,(s-1)-K_0]t^{-2} = \frac{(8\pi+2\mu)\,I^2t^{(4p_1)}}{8\pi} +2\mu p+\mu\rho+(8\pi+3\mu)\,\mu\lambda, \quad (16)$$

$$[-\frac{1}{2}\,(\theta-s^2)]t^{-2} = -\frac{(8\pi+2\mu)\,I^2t^{(4p_1)}}{8\pi} +2\mu p+(8\pi+3\mu)\,\rho+\mu\lambda. \quad (17)$$

From Eqs. (16)-(17), we obtain the following expressions for the string tension density, rest energy density, pressure and particle energy density:

$$\lambda = \frac{(p_2-p_1)\,(s-1)\,t^{-2}}{(8\pi+2\mu)} -\frac{I^2t^{(4p_1)}}{4\pi}, \quad (18)$$

$$\rho = \frac{[p_1\,(1-s)+\theta-s]t^{-2}}{(8\pi+2\mu)} -\frac{I^2t^{(4p_1)}}{4\pi}, \quad (19)$$

$$p = [\frac{d_1}{t^2}] +[\frac{(4\pi-\mu)\,I^2t^{(4p_1)}}{8\pi\mu}], \quad (20)$$

$$\rho_p = [\frac{(\theta-s)-p_2\,(s-1)}{8\pi+2\mu}]t^{-2}, \quad (21)$$

where:

$d_1 = [\frac{d_{11}-d_{12}-d_{13}+d_{14}}{2\mu(8\pi+2\mu)}]d_{11} = (s-1)\,(8\pi+\mu)\,p_1\,,$
$d_{12} = (s-1)\,\mu p_2\,,$
$d_{13} = (4\pi+\mu)\,s^2\,,$
$d_{14} = (8\pi+\mu)\,s-(4\pi+\mu)\,\theta\,.$

From Eq. (21), we found that the energy density of particle is independent of magnetic field. The plots of λ, ρ, ρ_p, w are shown in Figs. (1), (2), (3) and (4). We must emphasize here that one may get different conclusions by choosing different values of constants. The graphs are plotted against redshift. It also gives size of the Universe at a particular time, here a higher z implies smaller Universe size in the past. The value of z is greater than 1 in the early era of the Universe and tends to be zero at the latter time ($z \leq 0$). The magnetic field is subtracted in expressions of the string tension density and rest energy density. The energy condition $\rho \geq 0$ is satisfied for $p_1 < 0$. We see that as $I > 0.5$ the sign of the rest energy density and string tension density is negative in the early era of the Universe and they approach zero at the later time. In this case, $\rho < 0$, and it is not of our interest. For $0 < I < 0.5$ they are positive and larger at initial stages and tend to zero at large time. Thus, the Universe is dominated by the particle energy density throughout the evolution of the Universe. Letelier (1983) has pointed out that the string tension density may be positive or negative. The negative value of the string tension density represents the disappearance of the string phase of the Universe. In other words, the Universe contains an anistropic fluid of particles.

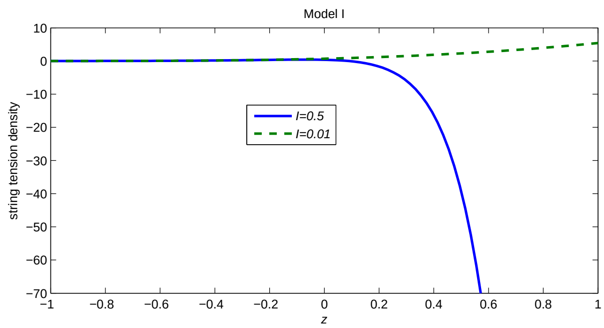

Fig. 1. *Variation of string tension density as a function of red shift.*

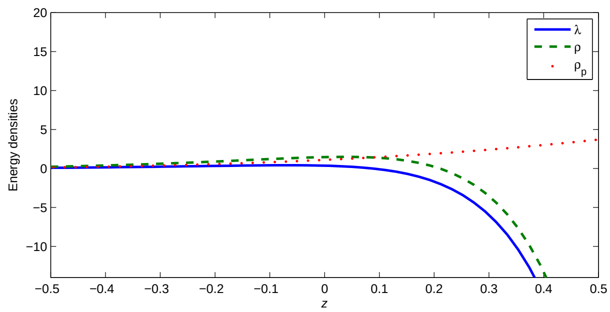

Fig. 2. *Variation of energy densities as a function of redshift.*

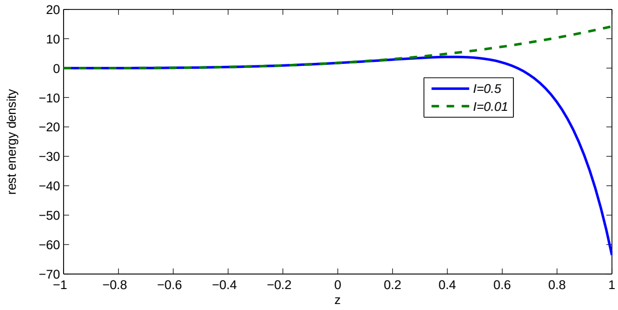

Fig. 3. *Variation of rest energy density as a function of redshift.*

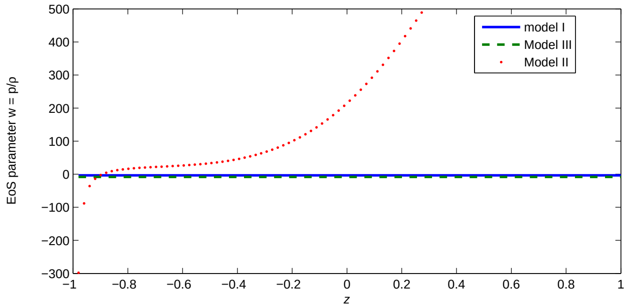

Fig. 4. *Variation of the EOS parameter as a function of redshift.*

Kibble (1976) stated that when the number of strings happens to be large, they govern the earlier history of the Universe. Strings can interact with matter leading to significantly local inhomogeneties. Pradhan et al. (2012) have observed the oscillation of λ initially and stability of λ with increasing time in case of the Bianchi type I metric in the general relativity. In the $f(R,T)$ theory, Sahoo et al. (2016a) have found that $\lambda = 0$ for the Bianchi type III spacetime and string exists in case of the Bianchi type VI0. In our model, the string network exists in the early Universe and eventually disappears which is in agrement with Kibble (1976).

We have noted that the string tension density and rest energy density vary for small and large value of I at the early stages of evolution of the Universe and at the later time, the effect of magnetic field disappears. The EoS parameter is given by relation between pressure and energy. The EoS parameter is considered as a function of time to explore the anisotropic dark energy models. It is given by the relation $\omega = p/\rho$. The possibility of the EoS parameter $\omega < -1$ is allowed. Frampton (2003) and Gil et al. (2002) have interpreted that the dark energy with $\omega < -1$ comes from the string theory, as closed strings on a toroidal cosmology. In our model, the EoS parameter is $\omega < -1$, thus we conclude that this variation of the EoS parameter comes from the string.

4. MODEL II

In this case, we explore the second model proposed by Harko et al. (2011):

$$f(R,T) = f_1(R) + f_2(T), \qquad (22)$$

where $f_1(R)$ is a function of R and $f_2(T)$ a function of T. The addition of the stress energy momentum tensor dependent function increases the possibility of modification of evolution of the Universe. The field equations using Eq. (22) take the following form:

$$\begin{aligned} & f'(R) R_{\mu\nu} - \frac{1}{2} f_1(R) g_{\mu\nu} \\ & + \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right) f_1'(R) \\ & = 8\pi T_{\mu\nu} - f_2' T_{\mu\nu} - f_2'(T)\theta_{\mu\nu} \\ & + \frac{1}{2} f_2(T) g_{\mu\nu}. \end{aligned} \qquad (23)$$

We assume the following functional form which is widely discussed in the $f(R)$ theory of gravitation. In the $f(R)$ gravity it has been shown that addition of an R^2 term to the Einstein gravitational action leads to a consist modified theory which may pass solar system test and is free from instability problem. It admits early inflation. Starobinsky (2007) proposed the model $f(R) = R + \alpha R^2$ which leads to acceleration and expansion of the Universe. Jamil et al. (2012) have investigated various cosmological models for a particular function $f(R,T) = R^2 + f(T)$. Here, we intend to consider the model given by:

$$f_1(R) = R + bR^m, b > 0, m > 0. \qquad (24)$$

We would like to stress that the choice of the function in the Eq. (24) may make the model consistent with the present state of the Universe. Further, we have chosen the following function:

$$f_2(T) = \mu T \qquad (25)$$

using the per-requisite Eqs. (1), (2), (22) and (25), so that the system reduces to the following field equations:

$$\begin{aligned} & [2p_1(s-1) - (s^2 - 2s + \theta)]t^{-2} \\ & -K_1 t^{-2m} + K_2[p_1 + (2m+2)p_1 + K_3] \\ & t^{(-2m-4)} = [-K_4 t^{2p_1+2p_3}] \\ & +2\mu P + \mu\lambda + \mu\rho, \end{aligned} \qquad (26)$$

$$\begin{aligned} & [2p_2(s-1) - (s^2 - 2s + \theta)]t^{-2} \\ & -K_1 t^{-2m} + K_2[p_2 + (2m+2)p_2 \\ & +K_3]t^{(-2m-4)} \\ & = [K_4 t^{2p_1+2p_3}] + 2\mu P + (16\pi - 5\mu) \\ & \lambda + \mu\rho, \end{aligned} \qquad (27)$$

$$\begin{aligned} &[2p_3(s-1)-(s^2-2s+\theta)]t^{-2} \\ &-K_1t^{-2m}+K_2[p_3+(2m+2)p_3+K_3] \\ &t^{(-2m-4)}=[-K_4t^{2p_1+2p_3}]+2\mu P \\ &+\mu\lambda+\mu\rho, \end{aligned} \tag{28}$$

$$\begin{aligned} &[\theta-s^2]t^{-2}-K_1t^{-2m} \\ &+K_2[\theta-s-2m-2]t^{-2m-4} \\ &=[K_4t^{2p_1+2p_3}]+2\mu P \\ &+2\mu\lambda+(16\pi+3\mu)\rho, \end{aligned} \tag{29}$$

where:

$$\begin{aligned} K_1 &= b\left(s^2-2s+\theta\right)^m, \\ K_2 &= 2bm\left(s^2-2s+\theta\right)^{m-1}, \\ K_3 &= -2m-2+(2m+3)(2m+2), \\ K_4 &= \frac{(16\pi+2\mu)I^2}{8\pi}. \end{aligned}$$

From Eqs. (26) and (28), we get $p_1 = p_3$. Using this relation, the above system of equations reduces to the following equations:

$$\begin{aligned} &[2p_1(s-1)-(s^2-2s+\theta)] \\ &t^{-2}-K_1t^{-2m}+K_2[p_1+(2m+2)p_1 \\ &+K_3]t^{-2m-4}=[-K_4t^{4p_1}]+2\mu P \\ &+\mu\lambda+\mu\rho, \end{aligned} \tag{30}$$

$$\begin{aligned} &[2p_2(s-1)-(s^2-2s+\theta)] \\ &t^{-2}-K_1t^{-2m}+K_2[p_2+(2m+2)p_2 \\ &+K_3]t^{(-2m-4)}=[K_4t^{4p_1}]+2\mu P \\ &+(16\pi-5\mu)\lambda+\mu\rho, \end{aligned} \tag{31}$$

$$\begin{aligned} &[\theta-s^2]t^{-2}-K_1t^{-2m} \\ &+K_2[\theta-s-2m-2]t^{-2m-4} \\ &=[K_4t^{4p_1}]+2\mu P+2\mu\lambda \\ &+(16\pi+3\mu)\rho. \end{aligned} \tag{32}$$

Using Eqs. (30)-(32), we obtain the following expressions for the rest energy density, string tension density and particle energy density:

$$\rho = K_5t^{-2}-K_6t^{-2m-4}-\frac{I^2t^{4p_1}}{4\pi}, \tag{33}$$

$$\lambda = K_7t^{-2}-K_8t^{-2m-4}-[\frac{(8\pi+\mu)I^2t^{4p_1}}{4\pi(3\mu-8\pi)}], \tag{34}$$

$$\rho_p = C_1t^{-2}+C_2t^{-2m-4}+C_3t^{4p_1}, \tag{35}$$

where:

$$\begin{aligned} K_5 &= [\frac{[-p_1(s-1)-s+\theta]t^{-2}}{8\pi+\mu}], \\ K_6 &= [\frac{bm\left(s^2-2s+\theta\right)^{m-1}}{8\pi+\mu}] \\ &[p_1+(2m+2)p_1 \\ &+(2m+2)(2m+3)+s-\theta], \\ K_7 &= \frac{[(-p_1-p_2)(s-1)]t^-2}{3\mu-8\pi}, \\ K_8 &= [\frac{bm\left(s^2-2s+\theta\right)^{m-1}}{3\mu-8\pi}] \\ &[p_1-p_2+(2m+2)(p_1-p_2)], \end{aligned}$$

$$\begin{aligned} C_1 &= \frac{C_{11}+C_{12}+C_{13}}{(8\pi+\mu)(3\mu-8\pi)}, \\ C_{11} &= (16\pi-2\mu)p_1(s-1) \\ C_{12} &= p_2(8\pi+\mu) \\ C_{13} &= (\theta-s)(3\mu-8\pi) \\ C_2 &= [\frac{K_2}{(16\pi+2\mu)(3\mu-8\pi)}] \\ &[p_1(16\pi-2\mu)(2m+3) \\ &-p_2(8\pi+\mu)(2m+3) \\ &+(\theta-s)(3\mu-8\pi) \\ &-(2m-2)(2m+3)(3\mu+8\pi)], \\ C_3 &= \frac{(16\pi-2\mu)I^2}{4\pi(3\mu-8\pi)}, \end{aligned}$$

From expressions (33)-(35), we see that ρ, λ and ρ_p are decreasing functions of time. The energy condition is satisfied for $p_1 < 0$. By considering particular values of the constants, plots of ρ, λ, ρ_p against redshift are shown in Figs. (5), (6), (7) and (8). We observe that when $m = 2$ and $I = 2$, at the initial stages $\lambda \leq 0, \rho \geq 0, \rho_p \geq 0$ i.e. the fact that the Universe is dominated by an anisotropic fluid particles. Also the string phase of the Universe disappears. Moreover, for $z \longrightarrow -1$, the string, energy and particle densities tend to zero: $\lambda,\ \rho,\ \rho_p \longrightarrow 0$. When $m = 0.5$ and $I = 2$ at the early stages of evolution of the Universe, the string and particle density become positive whereas $\rho \leq 0$ and, as $z \longrightarrow -0$, $\lambda, \rho, \rho_p \longrightarrow 0$. Thus, a string network exists in the early history of the Universe and it disappears in the future time but, as $\rho < 0$, this case is not of our interest.

For $m = 0.5$ and $I = 0.01$, one has $\lambda \geq 0$, $\rho \geq 0$ at initial stages of the Universe, therefore the string exists in the early stages of the Universe and, at later time, it eventually vanishes. It is interesting to note that $\lambda \leq 0$ when $m = 2$, and $\lambda \geq 0$ for $m = 0.5$. When $I = 2$, $\rho \leq 0$ whereas for $I = 0.01$, $\rho \geq 0$. λ changes with the changing value of m whereas ρ changes with I. The existence of string may depend on R^m in Eq. (25) whereas the magnetic field may have effect on the rest energy density. The value of the EoS parameter is much larger than 1 which may be the effect of curvature. Our model is consistent with the argument of Kibble (1976) for $m = 0.5$, $I = 0.01$.

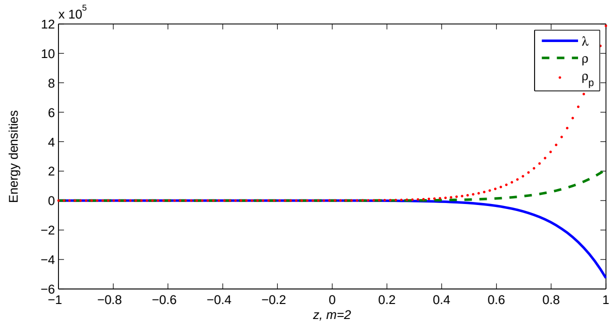

Fig. 5. *Variation of energy densities for m=2 as a function of redshift.*

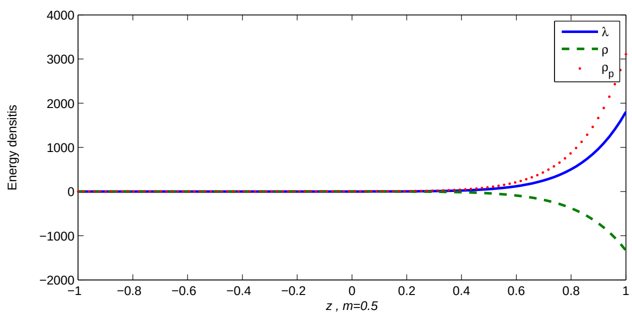

Fig. 6. *Variation of energy densities for m=0.5 as a function of redshift.*

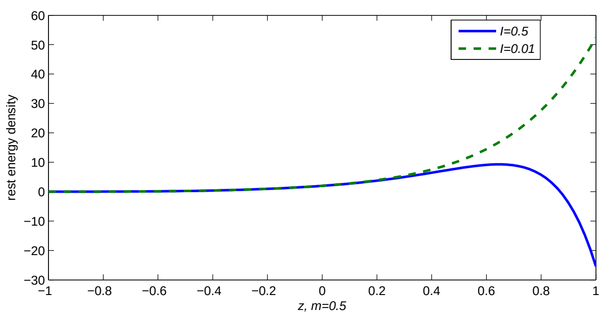

Fig. 7. *Variation of rest energy density for m=0.5 as a function of redshift.*

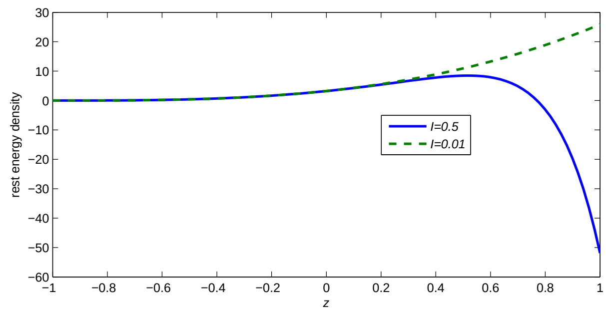

Fig. 8. *Variation of the rest energy density for m=2 as a function of redshift.*

5. MODEL III

Herein, we deal with the study of the function given by:

$$f(R,T) = \mu R + \mu T. \tag{36}$$

This type of functional conjecture gives us the effect of the cosmological constant Λ which depends on the stress energy tensor. We call this the effective cosmological constant. These types of model for the Kaluza-Klein space time is discussed by Sahoo et al. (2016). Poplawski (2006) have presented the cosmological constant as a function of the trace energy momentum tensor. Using Eq. (36), the field equations of the modified $f(R,T)$ gravity are obtained as:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu}{\mu}\right)T_{ij} + \left(P + \frac{1}{2}T\right)g_{ij}. \tag{37}$$

The well known field equations of general relativity with the cosmological constant are given by:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} + \Lambda g_{ij}. \tag{38}$$

Comparing Eqs. (37) and (38), we get $\Lambda = \Lambda(T) = P + \frac{1}{2}T$ and $-8\pi = \frac{8\pi+\mu}{\mu}$. Thus, we have:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu}{\mu}\right)T_{ij} + \Lambda g_{ij}. \tag{39}$$

Using Eqs. (1) and (2), we have the following field equations:

$$[p_1(s-1) - K_0]t^{-2} = \left(\frac{8\pi + \mu}{\mu}\right)\left(\frac{-I^2 t^{(2p_1+2p_3)}}{8\pi}\right) + \Lambda, \tag{40}$$

$$[p_2\,(s-1) - K_0]t^{-2} = \left(\frac{8\pi+\mu}{\mu}\right)\left(\lambda + \frac{I^2 t^{(2p_1+2p_3)}}{8\pi}\right) + \Lambda, \tag{41}$$

$$[p_3\,(s-1) - K_0]t^{-2} = \left(\frac{8\pi+\mu}{\mu}\right)\left(\frac{-I^2 t^{(2p_1+2p_3)}}{8\pi}\right) + \Lambda, \tag{42}$$

$$[\frac{1}{2}(\theta - s^2)]t^{-2} = (\frac{8\pi+\mu}{\mu})(\frac{I^2 t^{2p_1+2p_3}}{8\pi} + \rho) + \Lambda. \tag{43}$$

From Eqs. (40) and (42), we obtain $p_1 = p_3$. Using this relation, the system of the above equations reduces to the following form:

$$[p_1\,(s-1) - K_0]t^{-2} = \left(\frac{8\pi+\mu}{\mu}\right)\left(\frac{-I^2 t^{(4p_1)}}{8\pi}\right) + \Lambda, \tag{44}$$

$$[p_2\,(s-1) - K_0]t^{-2} = \left(\frac{8\pi+\mu}{\mu}\right)\left(\lambda + \frac{I^2 t^{(4p_1)}}{8\pi}\right) + \Lambda, \tag{45}$$

$$[\frac{1}{2}\,(\theta - s^2)]t^{-2} = \left(\frac{8\pi+\mu}{\mu}\right)\left(\frac{I^2 t^{4p_1}}{8\pi} + \rho\right) + \Lambda. \tag{46}$$

Using Eqs. (44)-(46), we obtain the rest density, string tension density, pressure and particle energy density as follows:

$$\rho = -\frac{\mu}{8\pi+\mu}[p_1\,(s-1) - \theta + s]t^{-2} - \frac{I^2 t^{4p_1}}{4\pi}, \tag{47}$$

$$\lambda = \frac{\mu\,(p_2 - p_1)\,(s-1)}{8\pi+\mu}t^{-2} - \frac{I^2 t^{4p_1}}{4\pi}, \tag{48}$$

$$P = \frac{K_9}{t^2} + \frac{(8\pi+3\mu)\,I^2 t^{4p_1}}{8\pi\mu}, \tag{49}$$

$$\rho_p = -\mu[\frac{p_2\,(s-1) + s - \theta}{8\pi+\mu}]t^{-2}, \tag{50}$$

where:

$K_9 = \frac{k_{91}-k_{92}+k_{93}}{(16\pi+2\mu)}.k_{91} = (8\pi+\mu)\left(\theta - s^2\right)$
$k_{92} = \mu\,(p_2 - p_1)\,(s-1)$
$k_{93} = (16\pi + 3\mu)\,(p_1\,(s-1) - \theta + s)$.

In this model, the energy density of particle is independent of magnetism. The energy condition $\rho \geq 0$ is satisfied for $p_1 < 0$. The rest energy density and string tension density are positive for $0 \leq I \leq 0.01$ and $z \geq 0.2$, and approach zero as $z \longrightarrow -1$. It is also observed that the string tension density and rest energy density are negative for $z \geq 0, I \geq 0.5$ and they approach zero as $z \longrightarrow -1$. The particle energy density is positive when $z \geq -0.2$ (see Figs. (9) and (10)). It is not independent of the term I. Thus, the particle energy density pervades the Universe. A close study of the expressions (48) and (49) reveals that the rest energy density and string tension density change with changing magnetic field. For large value of I, the string phase of the Universe disappears. The EoS parameter is less than -1 for small values of I. This may be the effect of string. The evolution of the cosmological constant is as shown in Fig. (11). The value of the cosmological constant is assumed to be positive. The observed value of Λ is approximately $10^{-10}(eV)^4$. It is essential to point out that the value of Λ was large during the early stages of the Universe. The expansion of the Universe was strongly influenced by the cosmological constant. It has been suggested that Λ depends on the Higgs scalar field. A positive cosmological constant means repulsive forces between galaxies whereas a negative cosmological constant connotes attractive forces between galaxies. A positive value of the cosmological constant balances the gravitational force and accelerates the expansion of the Universe. A negative value of the cosmological constant represents the ordinary matter which decelerates the Universe. In our derived model, Λ is negative and much smaller than -1 at initial stages of the Universe and the effect of Λ at later time tends to zero. Thus, we find that the Universe is decelerating at the initial stages of the Universe where the string exists.

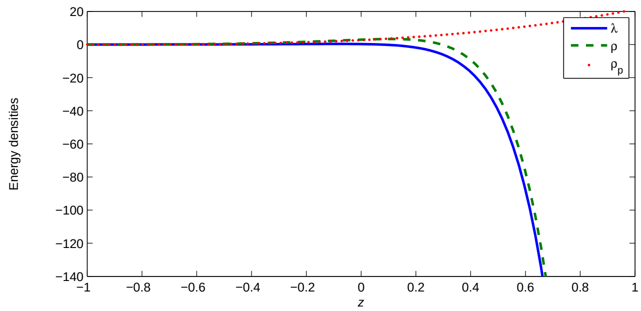

Fig. 9. *Variation of energy densities as a function of redshift.*

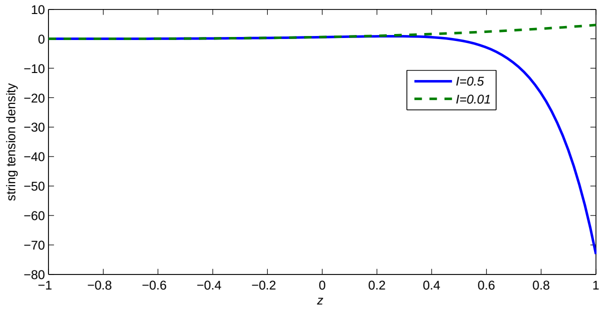

Fig. 10. *Variation of rest energy density as a function of redshift.*

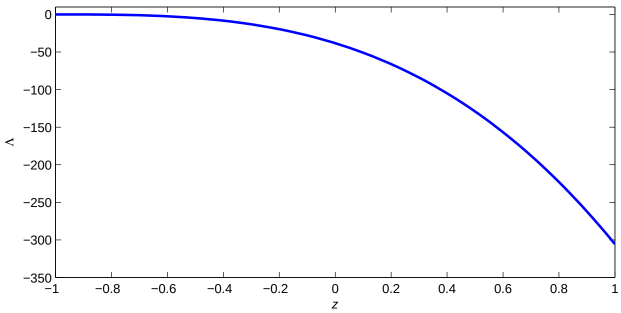

Fig. 11. *Variation of cosmological constant against redshift.*

We have the following physical parameters for the above three models: volume

$$V = t^{2p_1+p_2}, \tag{51}$$

expansion scalar

$$\theta_1 = 3H = \frac{2p_1 + p_2}{t}, \tag{52}$$

deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{3 - 2p_1 - p_2}{2p_1 + p_2}. \tag{53}$$

Keeping in mind the restriction of the constant $p_1 < 0$, we see that the specific volume is increasing function of time t for $p_2 > -2p_1$. It is zero at $t = 0$. The expansion scalar θ_1 is decreasing function of time t. It is large near Big bang singularity. The rate of expansion of the Universe decreases with increasing time.The deceleration parameter is constant throughout the evolution of the Universe. The sign of the deceleration parameter indicates whether the Universe is accelerating or decelerating. The Universe is decelerating at the past and accelerating at the present. Here, the deceleration parameter is negative for $P_2 > -2p_1 + 3$. Thus, the Universe is accelerating and expanding for $p_2 > -2p_1 + 3$ which is consistent with the cosmological observations.

6. CONCLUSION

In the presented research study, we have explored the Kasner space-time in the presence of the string and magnetic field in context of the modified $f(R,T)$ theory of gravitation. We have found that:

1. In model I, the Universe is dominated by the particle energy density. For large values of constant$I$ the rest energy density and string tension density are negative near $z = 1$ and approach zero as z tends to -1.
2. In model II, the energy densities are affected by the curvature term as well as magnetic field.
3. In model III, the particle energy density is independent of the term I. The rest energy density and string tension density are positive for small values of I near $z = 1$.
4. The Universe is expanding and accelerating. It is consistent with the observational data.
5. The EoS parameter is less than -1 which may be an effect of the string in Models I and III. In Model III, the EoS parameter is positive and larger than 1 which may be an effect of the curvature term in the function.
6. The cosmological constant term is negative throughout the evolution of the Universe.
7. Geometry of the space-time remains same for the different functional forms of $f(R,T)$, but the matter distribution is slightly changed.
8. The component of magnetic field reduces the rest energy density and string tension density.

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REFERENCES

Adhav, K. S.: 2012, *Astrophys. Space Sci.*, **339**, 365.
Bali, R. and Dave, S.: 2001, *Pramana J. Phys.*, **56**(4), 513.
Bamba, K. et al.: 2012, *Astrophys. Space Sci.*, **342**, 155.
Banerjee, A., Sanyal, A. K. and Chakroborty, S.: 1990, *Pramana J. Phys.*, **34**, 1.
Bastero-Gil, M., Frampton, P. H. and Mersini, L.: 2002, *Phys. Rev. D.*, **65**, 106002.
Camanho, X. O., Dadhich, N. and Molin, A.: 2016, arXiv:1604.01054v1
Capozziello, S. et al.: 2008, *Phys Lett. B*, **664**, 12.
Cataldo, M. and Campo, S. del: 2002, *Revist. Mex. Fisical*, **48**(3), 84.
Chiba, T., Smith, L. and Erickcek, A. L.: 2007, *Phys. Rev. D.*, **75**, 124014.
Clifton, T.: 2006, arXiv:gr-qc/0607096v2
Clifton, T. and Barrow, J. D.: 2005, arXiv:gr-qc/0509059v2
Clifton, T. and Barrow, J. D.: 2006, arXiv:gr-qc/0601118v1
Frampton, P. H.: 2003, *Phys. Lett. B*, **555**, 139.

Frampton, P. H.: 2005, arXiv:0506676v1
Gao, C. and Shen, Y.: 2016, arXiv:1602.08764v6
Gil, M. B., Frampton, P. H. and Mersini, L.: 2002, arXiv:hep-th/0110167v3
Harko, T., Lobo, F. S. N., Nojiri, S. and Odintsov, S. D.: 2011, *Phys. Rev. D.*, **84**, 024020.
Houndjo, M. J. S.: 2012, *Int. J. Mod. Phys. D.*, **21**, 125003.
Jamil, M. Momeni, D., Raza, M. and Myrzakulov, R.: 2012, *Eur. Phys. J. C*, **72**, 1999.
Katore, S. D.: 2015, *Int. J. Theor. Phys.*, **54**(8), 2700.
Katore, S. D. and Hatkar, S. P.: 2015, *Astrophys. Space Sci.*, **357**, 1.
Katore, S. D. and Hatkar, S. P.: 2016a, *Indian J. Phys.*, **90**(2), 243.
Katore, S. D. and Hatkar, S. P.: 2016b, *Prog. Theor. Exp. Phys.*, **33**, E01.
Katore, S. D., Hatkar, S. P. and Baxi, R. J.: 2016a, *Found. Phys.*, **46**(4), 409.
Katore, S. D., Hatkar, S. P. and Baxi, R. J.: 2016b, *Astrophys. J.*, **59**(4), 525.
Kibble, T. W. B.: 1976, *J. Phys. A. Math. Gen.*, **9**, 8.
Kibble, T. W. B., Vilenkin, A. and Shellard, E. P. S.: 1994, Cosmic string and other topological defects, Cambridge University press, Cambridge.
Letelier, P. S.: 1983, *Phys. Rev. D*, **28**, 2414.
Moraes, P. H. R. S.: 2014, *Astrophys. Space Sci.*, **352**, 273.
Multamaki, T. and Vilja, I.: 2007, *Phys. Rev. D.*, **76**, 064021.
Naidu, R. L., Naidu, K. D., Babu, K. S. and Reddy, D. R. K.: 2013, *Astrophys. Space Sci.*, **347**, 197.
Nojiri, S. and Odintsov, S. D.: 2007, *Int. J. Geom. Method. Mod. Phys.*, **4**, 115.
Paliathanasis, A.: 2016, arXiv:1512.03239v2
Paliathanasis, A., Said, J. L. and Barrow, J. D.: 2018, *Phys. Rev. D.*, **97**, 044008.
Pasqua, A., Chattopadhyay, S. and Khomenkoc, I.: 2013, *Cand. J. Phys.*, **91**, 632.
Poplawski, N. J.: 2006, arXiv:0608031
Pradhan, A., Kumhar, S. S. and Jotania, K.: 2012, *Pal. J. Math.*, **1**(2), 117.
Riess, A. G. et al.: 1998, *Astron. J.*, **116**, 1009.
Reddy, D. R. K. and Naidu, R. L.: 2007, *Int. J. Theor. Phys.*, **46**, 2788.
Reddy, D. R. K., Santhikumar, R. and Naidu, R. L.: 2012, *Astrophys. Space Sci.*, **342**, 249.
Sahoo, P. K., Mishra, B. and Tripathy, S. K.: 2016, *Ind. J. Phys.*, **94**(4), 485.
Sahoo, P. K, Mishra, B., Sahoo, P. and Pacif, S. K. J.: 2016a, *Eur. Phys. J. Plus*, **131**, 333.
Shamir, M. F.: 2010, *Astrophys. Space Sci.*, **330**, 183.
Sharma, N. K. and Singh, J. K.: 2014, *Int. J. Theor. Phys.*, **53**, 2912.
Shingh, C. P. and Singh, V.: 2014, *Gen. Relativ. Gravit.*, **46**, 1696.
Skugoreva, M. A. and Toporensky, A. V.: 2018, arXiv:1711.07069v4
Spergen, D. N. et al.: 2003, *Astrophys. J. Suppl.*, **148**, 175.
Stachel, J.: 1980, *Phys. Rev. D.*, **21**, 2171.
Starobinsky, A. A.: 2007, *J. Exp. Theor. Phys. Lett.*, **86**, 157.
Tegmark, M. et al.: 2004, *Phys. Rev. D.*, **69**, 103501.
Wainwright, J. and Krasinski, A.: 2008, *Gen. Relativ. Gravit.*, **40**, 865.
Yilmaz, I.: 2006, *Gen. Relativ. Gravit.*, **38**, 1397.
Zeldovich, Ya. B.: 1980, *Mon. Not. R. Astron. Soc.*, **192**, 663.

КОСМОЛОШКИ МОДЕЛИ КАЗНЕРОВОГ ТИПА СА МАГНЕТИЗОВАНИМ СТРУНАМА У $f(R,T)$ ТЕОРИЈИ ГРАВИТАЦИЈЕ

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Оригинални научни рад

Разматрана је метрика типа Бјанчи-I Казнер са космичким струнама и магнетним пољем у оквиру $f(R,T)$ теорије гравитације. За истраживање су изабрана три различита функционална облика функције $f(R,T)$. Пронашли смо да струне постоје у раним фазама еволуције Универзума, и нестају са проласком времена. Могуће је да се варијација параметра из једначине стања $\omega = p/\rho < -1$ јавља услед деловања струна. Нашли смо да се тензија струна и густина енергије мировања смањују при постојању магнетног поља. Универзум се шири и ширење се убрзава.